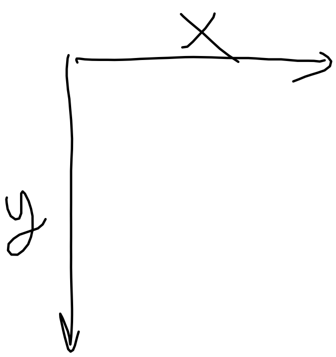
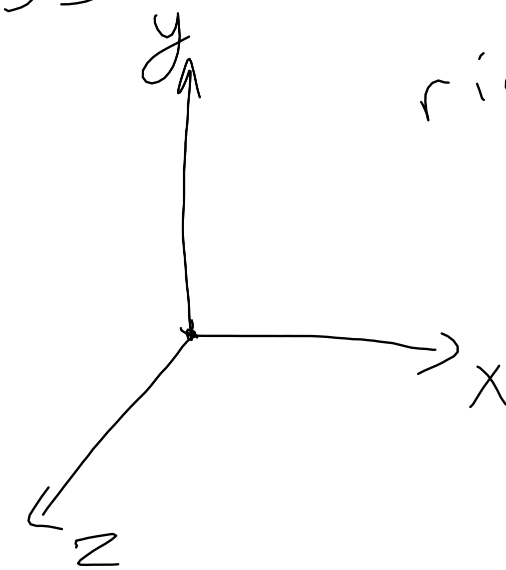
2D



3D

right-handed



Vector Operations

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$$\vec{a} = (a_x, a_y, a_z) \qquad \vec{b} = (b_x, b_y, b_z)$$

Dot product: $\vec{a} \cdot \vec{b}$

$$= a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$$

$$= \|a\| \cdot \|b\| \cdot \cos(\theta)$$

length

Cross product $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \left(a_y \cdot b_z - a_z \cdot b_y ,\ a_z \cdot b_x - a_x \cdot b_z ,\ a_x \cdot b_y - a_y \cdot b_x \right)$$

$$= \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin(\theta) \cdot \vec{n}$$

$\vec{n}$: perpendicular to the plane containing a, b
(unit)

Cross Product

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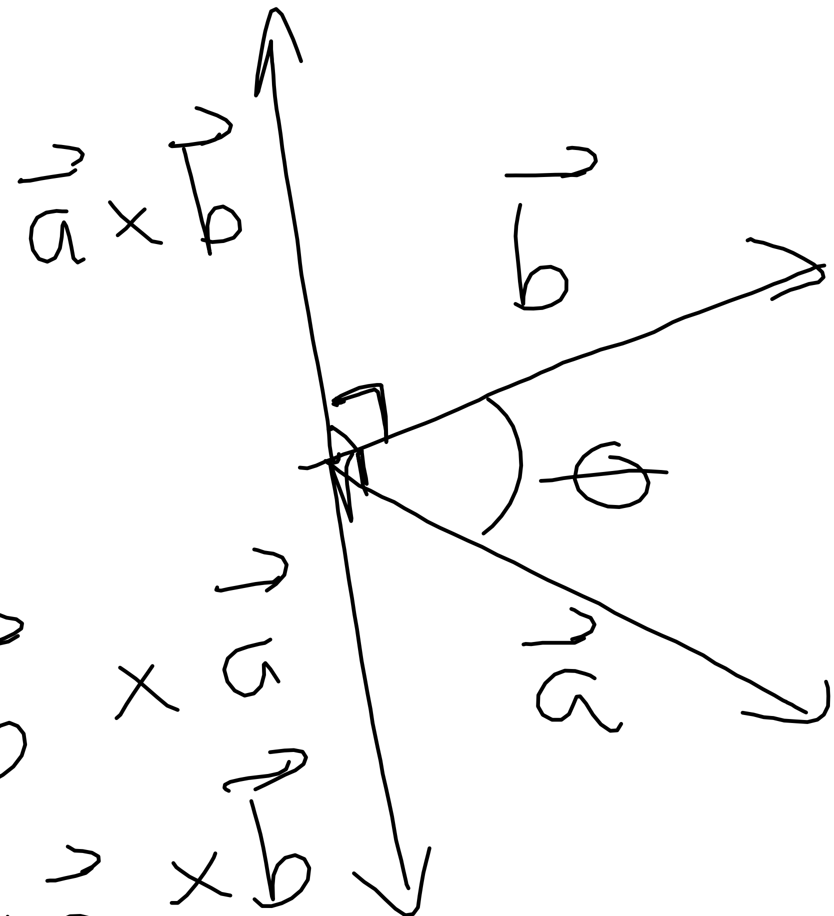

$\vec{b} \times \vec{a}$
$= -\vec{a} \times \vec{b}$

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3D Transforms

Translation

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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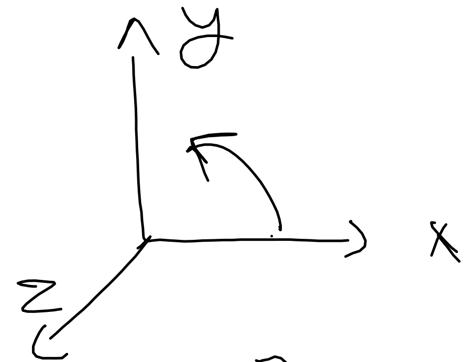
Rotation



around z-axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Rotation x-axis
from y to z

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Rotation y-axis from z to x

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate around parallel axis.

1) translate axis of rotation to origin
2) rotate around primary axis
3) translate back

Rotate around arbitrary axis

1. Translate so the axis goes through the origin
2. Rotate so axis becomes a primary axis
3. Do rotation around primary axis
4. Undo step 2
5. Undo step 1

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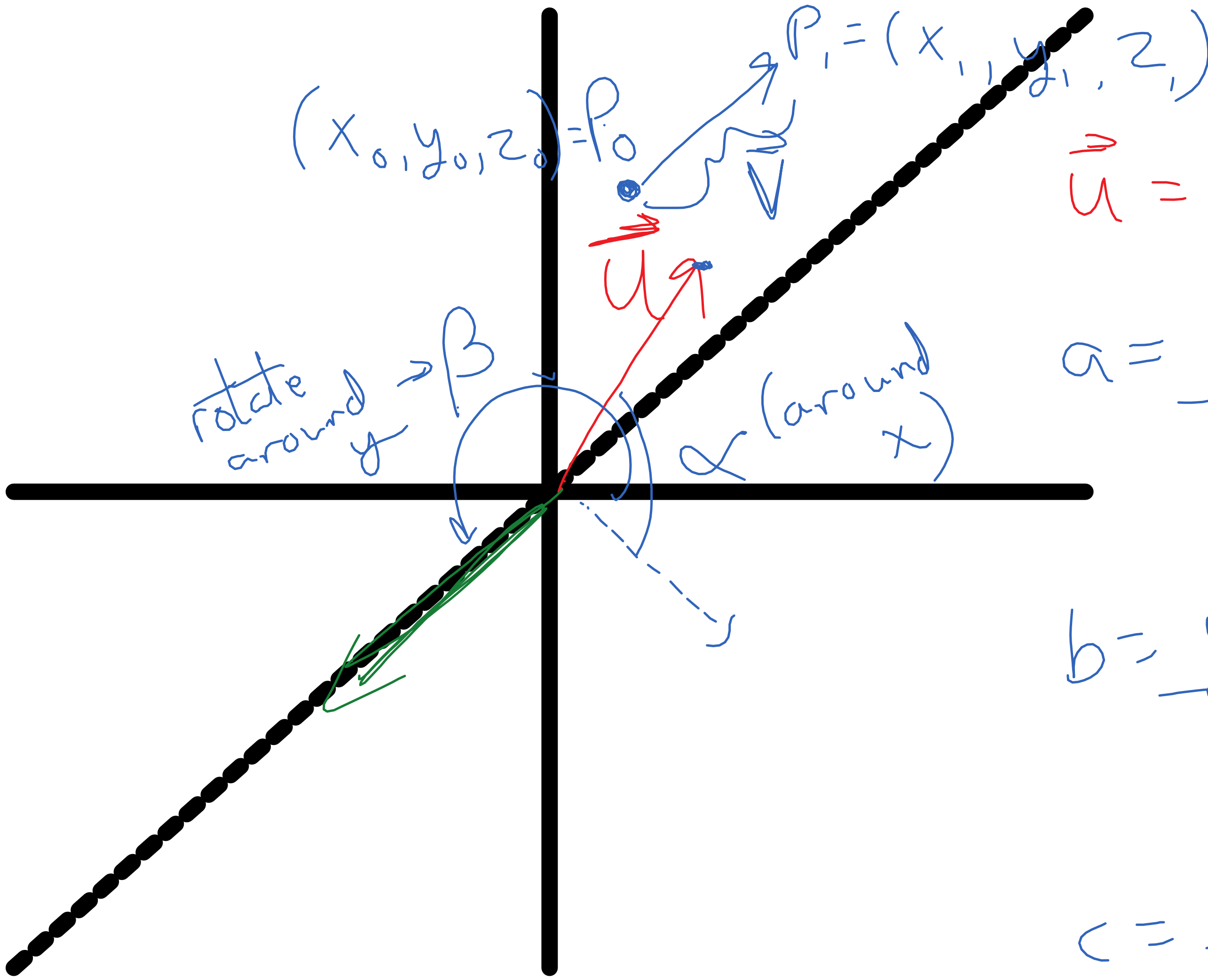


$$\vec{u} = (a, b, c)$$

$$a = \frac{x_1 - x_0}{\|\vec{V}\|}$$

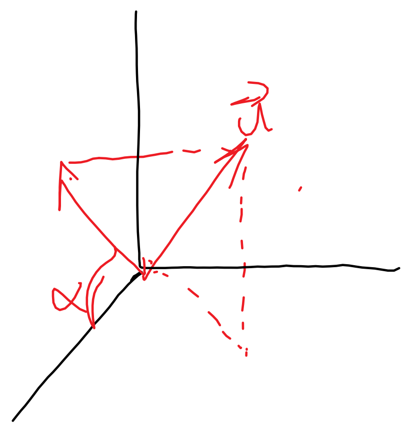
$$b = \frac{y_1 - y_0}{\|\vec{V}\|}$$

$$c = \frac{z_1 - z_0}{\|\vec{V}\|}$$

$$\|V\| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

find $\cos(\alpha)$ and $\sin(\alpha)$

α: angle of rotation to put $\vec{u}$ into xz plane



$\vec{u}'$ projection of $\vec{u}$ in yz plane

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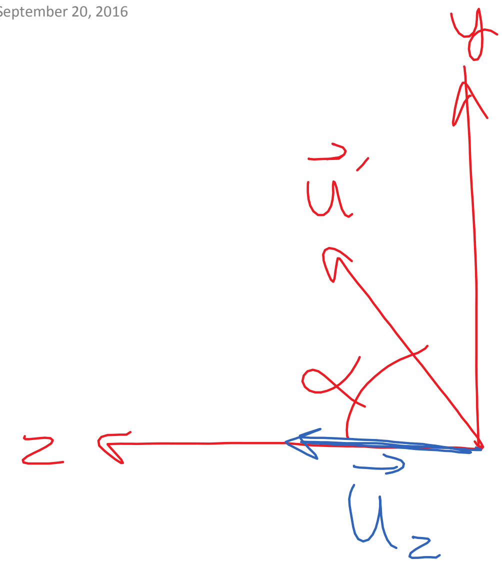

$$\vec{u}' \cdot \vec{u}_z = \|\vec{u}'\| \cdot \|\vec{u}_z\| \cdot \cos(\alpha)$$

$$\vec{u}_z = (0, 0, 1)$$

$$\cos(\alpha) = \frac{\vec{u}' \cdot \vec{u}_z}{\|\vec{u}'\| \cdot \underbrace{\|\vec{u}_z\|}_{1}} \leftarrow c$$

$$\vec{u}' = (0, b, c)$$

$$d = \sqrt{b^2 + c^2}$$

$$\cos(\alpha) = \frac{c}{d}$$

$$\vec{u'} \times \vec{u_2} = \|\vec{u'}\| \cdot \|\vec{u_2}\| \cdot \sin(\alpha) \cdot \cancel{\vec{n}}\ \vec{u_x}$$

$$\vec{u_x} \cdot \sin(\alpha) = \frac{\vec{u'} \times \vec{u_2}}{\underbrace{\|\vec{u'}\|}_{d} \cdot \underbrace{\|\vec{u_2}\|}_{1}} = \frac{b \cdot \vec{u_x}}{d}$$

$$\vec{u'} \times \vec{u_2} = (b-0,\ 0,\ 0) = b \cdot \vec{u_x}$$

$(0, b, c)$ $(0, 0, 1)$

so: $\sin(\alpha) = \dfrac{b}{d}$

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$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate back $-\alpha$

$$\cos(-\alpha) = \cos(\alpha)$$
$$\sin(-\alpha) = -\sin(\alpha)$$

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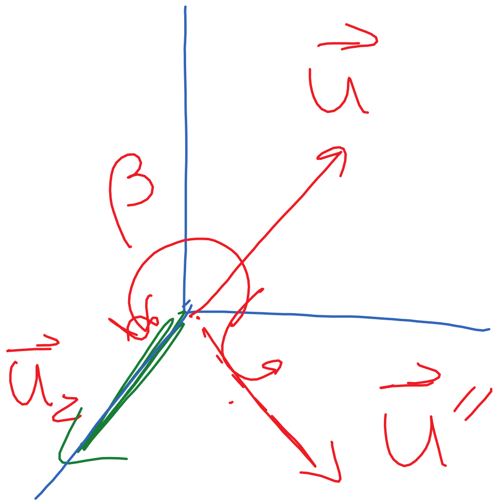

$\vec{u}'' = \vec{u}$ rotated into xz plane

$\vec{u}_z = (0, 0, 1)$

$\vec{u}'' = (a, 0, d)$ $\quad ||\vec{u}''|| = 1$

$\cos(\beta) = d$

$\sin(\beta) = -a$