Matrix Math

Thursday, September 15, 2016
2:33 PM

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 7 & 5 \end{bmatrix}$$

2×3

$m \times n$

rows columns

- multiply by scalar s $\quad s \cdot A$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & & b_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & & \\ & & \boxed{} \\ & & \end{bmatrix}$$

$$c_{ij} = a_{ij} + b_{ij}$$

Matrix Multiplication

Thursday, September 15, 2016
2:45 PM

$A: m \times n$ $\quad$ $A \cdot B : m \times r$

$B: n \times r$ $\quad$ $\cancel{B \cdot A}$ for $r \neq m$

$m \times n \cdot n \times 1 \Rightarrow m \times 1$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_1 + a_{12} \cdot b_2 + \cdots a_{1n} b_n \\ \vdots \\ a_{m1} \cdot b_1 + \cdots + a_{mn} \cdot b_n \end{bmatrix}$$

$$\underset{2 \times 3}{\begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}} \cdot \underset{3 \times 1}{\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}} = \begin{bmatrix} 4 + 25 + 8 \\ 6 + 30 + 9 \end{bmatrix} = \begin{bmatrix} 37 \\ 45 \end{bmatrix}$$

Thursday, September 15, 2016
2:57 PM

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & & \\ b_{n1} & \cdots & b_{nl} \end{bmatrix}$$

$m \times n$ $\qquad$ $n \times l$

$$= \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} + \cdots + a_{1n} b_{n1} \\ \\ \boxed{c_{ij}} \\ \\ \end{bmatrix}$$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} \cdot b_{2j} + \cdots$$
$$a_{in} \cdot b_{nj}$$

$$= \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

$$A = \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

2×3 $\qquad$ 3×2

$$A \cdot B = \begin{bmatrix} 2+15+40 & 4+20+48 \\ 3+18+45 & 6+24+54 \end{bmatrix}$$

2×2

$$= \begin{bmatrix} 57 & 72 \\ 66 & 84 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 8 & 17 & 26 \\ 18 & 39 & 60 \\ 28 & 61 & 94 \end{bmatrix}$$

3×3

Thursday, September 15, 2016
3:12 PM

point $\begin{bmatrix} x \\ y \end{bmatrix}$ transform to $\begin{bmatrix} x' \\ y' \end{bmatrix}$

Scaling $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x \cdot x \\ s_y \cdot y \end{bmatrix}$

Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

$$\begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

Translation $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$

$$\lfloor \quad \rfloor \qquad \lfloor ty \rfloor \qquad \lfloor y + \lfloor y \rfloor \rfloor$$

Thursday, September 15, 2016
3:17 PM

$(x, y) \longrightarrow (\quad , \quad , \quad)$

Homogeneous coord system

(x_h, y_h, h)

s.t. $x = \frac{x_h}{h}$ $\quad y = \frac{y_h}{h}$

$(x \cdot h, y \cdot h, h)$

set $h = 1$ $\quad (x, y, 1)$

Scale

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x \cdot x \\ s_y \cdot y \\ 1 \end{bmatrix}$$

Rot.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ 1 \end{bmatrix}$$

$$0 \qquad 0 \qquad 1 \quad 1 \qquad 1$$

Translate

$$\begin{bmatrix}1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x \\ y \\ 1\end{bmatrix} = \begin{bmatrix}x + t_x \\ y + t_y \\ 1\end{bmatrix}$$

Thursday, September 15, 2016
3:25 PM

Fixed pt. scaling (x_f, y_f)

1. move (x_f, y_f) to $(0,0)$
2. scale (S_x, S_y)
3. move $(0,0)$ to (x_f, y_f)

$$\underbrace{\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix}}_{3}\left(\underbrace{\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{2}\left(\underbrace{\begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}}_{1}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right)\right)$$

$$\begin{bmatrix} x - x_f \\ y - y_f \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x(x - x_f) \\ S_y(y - y_f) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} s_x(x - x_f) + x_f \\ s_y(y - y_f) + y_f \\ 1 \end{bmatrix}$$

$$A(B(C(D))) \qquad (ABC)(D)$$

$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f \\ 0 & s_y & y_f \\ 0 & 0 & 1 \end{bmatrix}$$

3   2   3, 2

$$(3,2) \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & \boxed{s_x(-x_f) + x_f} \\ 0 & s_y & \boxed{s_y(-y_f) + y_f} \\ 0 & 0 & 1 \end{bmatrix}$$

3, 2, 1

$$\begin{bmatrix} s_x & 0 & -s_x x_f + x_f \\ 0 & s_y & -s_y y_f + y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x s_x - s_x x_f + x_f \\ y s_y - s_y y_f + y_f \\ 1 \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 1 \end{matrix} \quad 1$$

$$\begin{bmatrix} S_x(x - x_f) + x_f \\ S_y(y - y_f) + y_f \\ 1 \end{bmatrix}$$

Thursday, September 15, 2016
3:44 PM

Identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$