

Matt Pennella

Suppose $A = \{u, v, x\}$ & $B = \{b\}$

a. show $A \times B$

$\{(u, b), (v, b), (x, b)\}$

b. what is power set of A

$\{\emptyset, \{u\}, \{v\}, \{x\}, \{u, v\}, \{u, x\}, \{v, x\}, \{u, v, x\}\}$

$$\boxed{7.2/17/}$$
$$f(x) = \frac{3x-1}{x}$$

$$\forall x \in \mathbb{R}, x \neq 0$$

Andy Krasny

For a function to be one-to-one, $f(x_1) = f(x_2)$ iff $x_1 = x_2$.

Proof Suppose $x_1 = x_2$. Then

$$\frac{3x_1-1}{x_1} = \frac{3x_2-1}{x_2}$$
$$x_2(3x_1-1) = x_1(3x_2-1)$$
$$3x_1x_2 - x_2 = 3x_1x_2 - x_1$$
$$\cancel{3x_1x_2} - \cancel{3x_1x_2} - x_2 = -x_1$$
$$-x_2 = -x_1$$
$$\underline{\underline{x_1 = x_2}}$$

therefore $f(x)$ is one-to-one.

□

61 # 11 Kirsten C

11. $A = \{0 < x \leq 2\}$

$B = \{1 \leq x < 4\}$

$C = \{3 \leq x < 9\}$

$A \cup B$

a. $0 < x \leq 2 \cup 1 \leq x < 4$

$0 < x < 4$

$A \cap B$

b. $1 \leq x \leq 2$

A^c

c. $0 \geq x \geq 2$

$A \cup C$

d. $0 < x \leq 2 \cup 3 \leq x < 9$

$A \cap C$

e. empty

B^c

f. $1 > x \geq 4$

$A^c \cap B^c$

g. $x < 1, x \geq 4, x \geq 0, x > 2$

$x \leq 0 \cup x \geq 4$

$A^c \cup B^c$

h. $x < 1, x > 2$

$(A \cap B)^c$

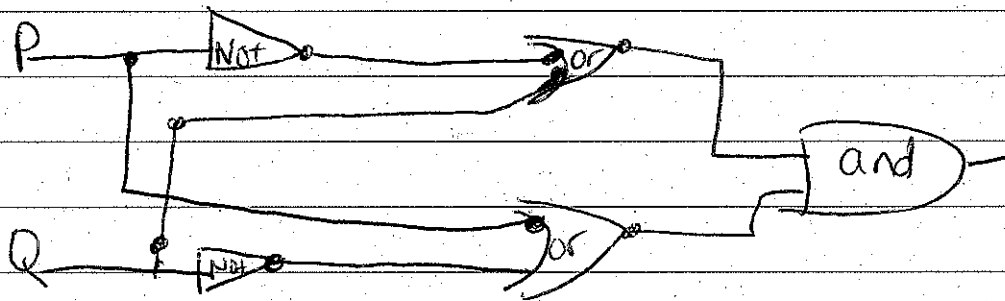
i. $x \leq 1, x > 2$

$(A \cup B)^c$

j. $x \leq 0 \cup x \geq 4$

James Ianro
CS 201 - Quiz

1) Design a circuit. What equation does the circuit create?



Answer:

$$\begin{aligned} &= (\sim P \vee Q) \wedge (\sim Q \vee P) \\ &\equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \\ &\equiv (P \leftrightarrow Q) \end{aligned}$$

by Demorgan's law.

Equation: $(P \leftrightarrow Q)$
Bi conditional.

Dan Minamora

10:00 MWF

Quiz 4

Prove by Mathematical Induction:

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{for all integers } n \geq 1$$

Solution:

$$P(n) \rightarrow 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall n \geq 1$$

$$\text{Basis: } P(1) \rightarrow 1^2 = 1 = \frac{1(1+1)(2+1)}{6} = \frac{6}{6} = 1 \quad \checkmark$$

$$\text{Ind. Hyp: Suppose } P(k) \text{ is true} \rightarrow 1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \quad \forall k \geq 1$$

[Show that $P(k+1)$ is true]

$$\begin{aligned} \hookrightarrow 1^2 + 2^2 + \dots + (k+1)^2 &= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \quad \forall k \geq 1 \\ &= \frac{(k+1)(k+2)(2k+3)}{6} \end{aligned}$$

Left side:

$$1^2 + 2^2 + \dots + (k+1)^2 = 1^2 + 2^2 + \dots + k^2 + (k+1)^2$$

$$\geq \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \text{by Ind. Hyp (Subst.)}$$

$$= \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

which is what we wanted to show

QED

~~$1^2 + 2^2 + \dots + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$~~
 ~~$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$~~
 ~~$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$~~
 ~~$= \frac{(k+1)(k+2)(2k+3)}{6}$~~

Tasker

Question:

In how many ways can the word "Permutation" be arranged, while keeping the letters 'p' and 'e' adjacent and the letters 'o' and 'n' adjacent?

Answer: $\underline{Pe} r m u t a t i o n \rightarrow 9!$

$e \underline{P} r m u t a t i o n \rightarrow 9!$

$\underline{Pe} r m u t a t i o n \rightarrow 9!$

$e \underline{P} r m u t a t i o n \rightarrow 9!$

$$\text{So, } 9! + 9! + 9! + 9! = 362880 + 362880 + 362880 + 362880$$

$$= 1451520 \text{ ways}$$

Answer To Review Question

~~$$b_n = 2n + 5$$~~

$$b_0 = 2$$

$$b_1 = 4 \cdot 2 + 5$$

$$b_2 = 4(4 \cdot 2 + 5) + 5 = 4^2 \cdot 2 + 4 \cdot 5 + 5$$

$$b_3 = 4(4(4 \cdot 2 + 5) + 5) + 5 = 4^3 \cdot 2 + 4^2 \cdot 5 + 4 \cdot 5 + 5$$

$$b_4 = 4(4(4(4 \cdot 2 + 5) + 5) + 5) + 5 = 4^4 \cdot 2 + 4^3 \cdot 5 + 4^2 \cdot 5 + 4 \cdot 5 + 5$$

$$b_n = 4^n \cdot 2 + \sum_{i=0}^{n-1} 5 \cdot 4^i$$

$$\sum_{i=0}^n r^i = \frac{r^{n+1} - 1}{r - 1}$$

$$b_n = 4^n \cdot 2 + 5 \cdot \sum_{i=0}^{n-1} 4^i$$

$$b_n = 4^n \cdot 2 + 5 \cdot \left(\frac{4^n - 1}{3} \right)$$