Transitivity

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Division into cases

$$
\begin{array}{l}
p \vee q \\
p \to r \\
q \to r \\
\hline
\therefore r
\end{array}
$$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

valid

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore\ p \end{array}$$

invalid

contradiction rule

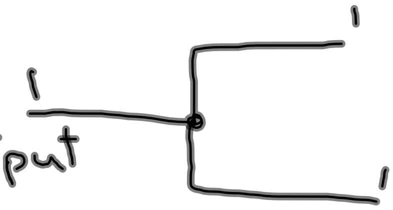
$\sim p \rightarrow c$

$\therefore p$

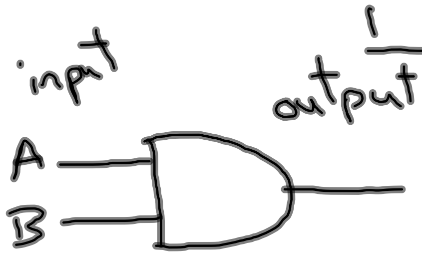
p	$\sim p$	c	$\sim p \rightarrow c$	
T	F	F	T	
F	T	F	F	

Digital logic circuits

Gates

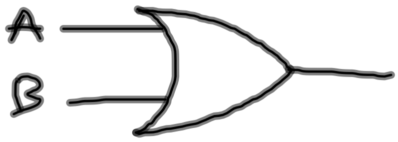


and

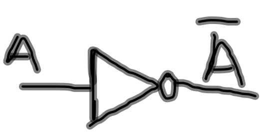


A	B	A and B
1	1	1
1	0	0
0	1	0
0	0	0

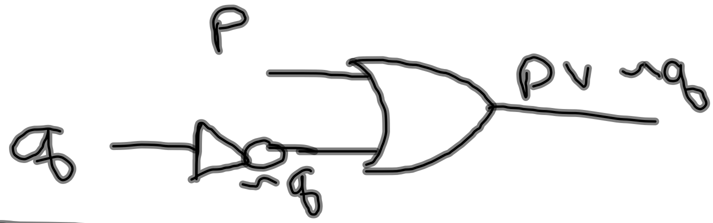
or



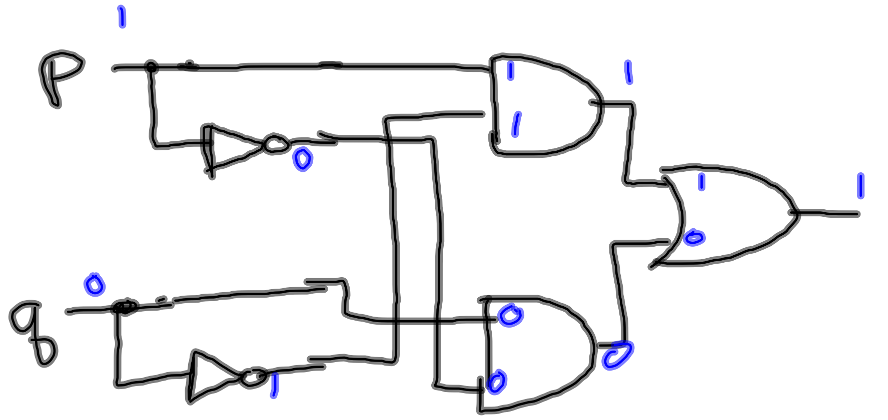
not

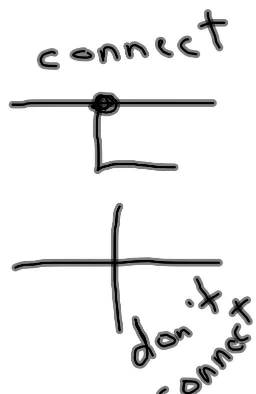


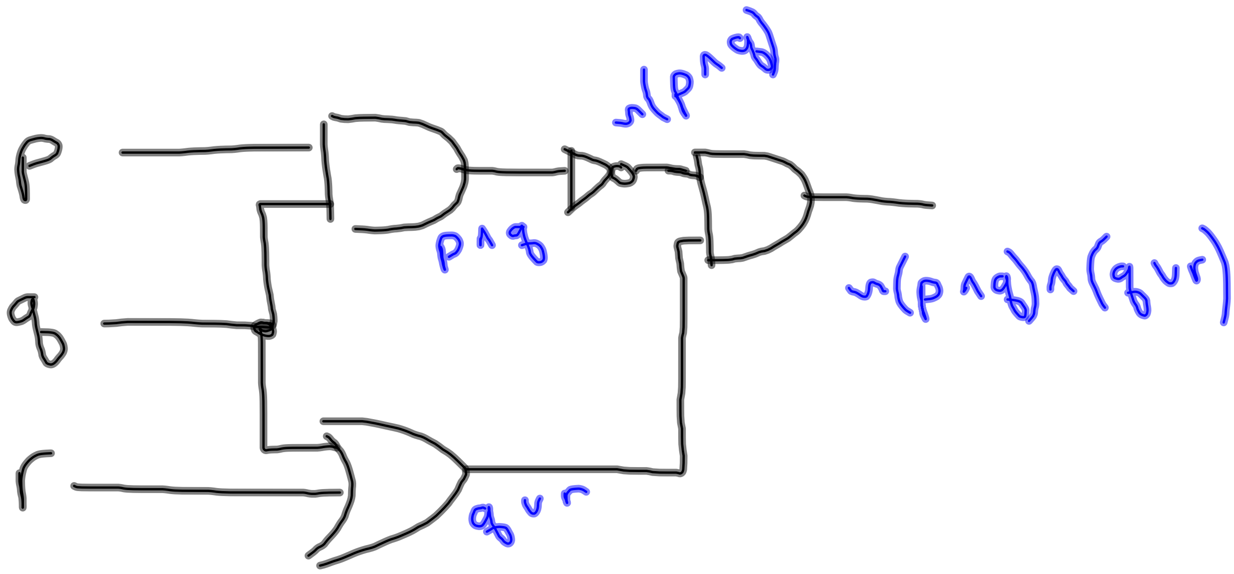
$p \vee \sim q$

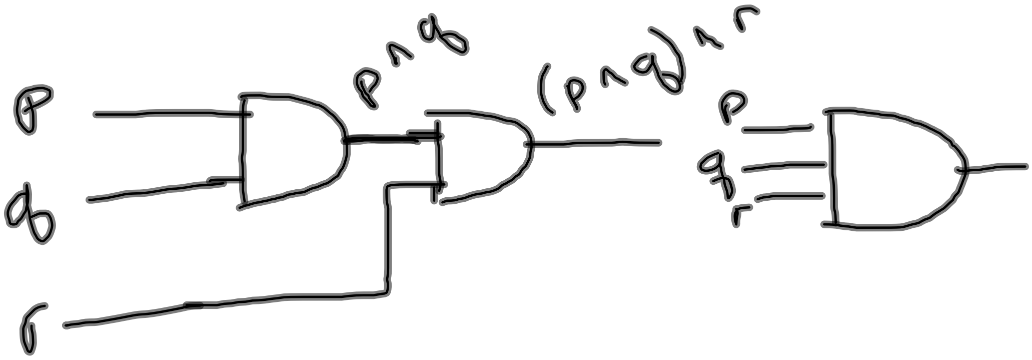


$(p \wedge \sim q) \vee (q \wedge \sim p)$






p
q
r
p ∧ q
~(p ∧ q)
~(p ∧ q) ∧ (q ∨ r)
q ∨ r


p
q
r
p ∧ q
(p ∧ q) ∧ r
p
q
r

p	q	r	f	a	b	c	d
1	1	1	1	1	0	0	0
1	1	0	0	0	0	0	0
1	0	1	1	0	1	0	0
1	0	0	0	0	0	0	0
0	1	1	1	0	0	1	0
0	1	0	1	0	0	0	1
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

$a = p \wedge q \wedge r$

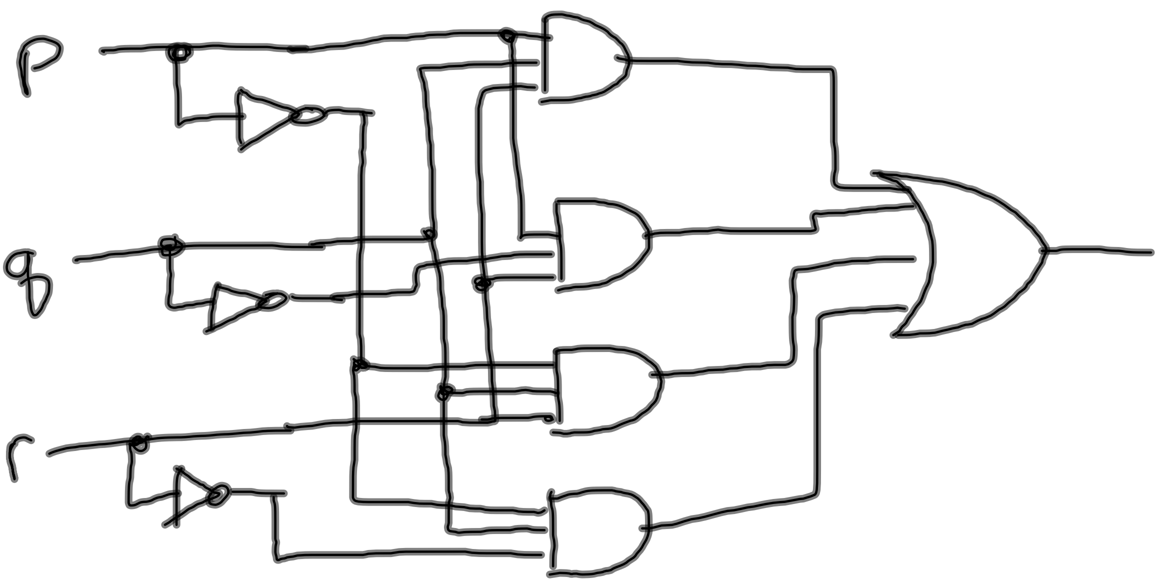
$b = p \wedge {\sim}q \wedge r$

$c = {\sim}p \wedge q \wedge r$

$d = {\sim}p \wedge q \wedge {\sim}r$

$$f = a \vee b \vee c \vee d$$

$$F = (p \wedge q \wedge r) \vee (p \wedge {\sim}q \wedge r) \vee ({\sim}p \wedge q \wedge r) \vee ({\sim}p \wedge q \wedge {\sim}r)$$

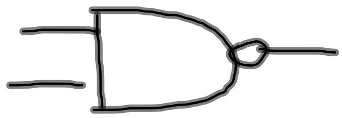


and, or and not - can be used to produce <u>any</u> Boolean formula. (computationally complete)

NAND - (not and)

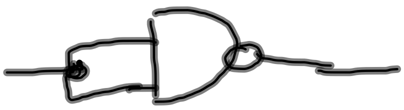
computationally complete

p	q	p nand q
1	1	0
1	0	1
0	1	1
0	0	1



Not (w/ nand)

$\sim(p \wedge p) \equiv \sim p$

p  $\sim p$