$$p \to q \equiv \sim p \lor q$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

negation of conditional

$$p \land \sim q$$

$$p \to q$$

raining get wet

$$\equiv \sim p \vee q$$

$$\sim q \to \sim p$$

contrapositive

$$\equiv \sim(\sim q) \vee \sim p$$

$$\equiv q \vee \sim p$$

p	q	$\sim p$	
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

converse of $p \to q$

$q \to p$

$\sim q \vee p$

p	q	$\sim q$	
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

inverse

$\sim p \to \sim q \equiv q \to p$

---

p only if q

$\sim q \to \sim p$

$p \to q$

Biconditional

- p if and only if q

p if q $\qquad q \to p$

p only if q $\qquad p \to q$

- $p \longleftrightarrow q \quad \equiv (p \to q) \wedge (q \to p)$
- p iff q

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

p : dinner

q : dessert

$$(p \to q) \wedge \cancel{(q \to p)}$$

$$(\sim p \Rightarrow \sim q)$$

arguments

If you eat dinner then you can have dessert.

You ate dinner.

Therefore You can have dessert

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

$$\xcancel{\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}}$$

$$\left.\begin{array}{l} p \to q \\ p \end{array}\right\} \text{premise}$$

$$\left.\therefore q\right\} \text{conclusion}$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

valid: whenever all premises are true, the conclusion is also true

$$\begin{array}{l} p \to q \\ q \\ \hline p \end{array}$$

not valid

p	q	$p \to q$
T	T	T
T	F	F
~~F~~	T	T
F	F	T

$p \wedge q$

$\therefore p$

valid

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \vee q$

$\therefore p$

not valid

p	q	$p \vee q$
T	T	T
T	F	T
~~F~~	T	T
F	F	F

$p \to q$

$r \to q$

$\therefore\ p \to r$

p	q	r	$p \to q$	$r \to q$	$p \to r$
T	T	T	T	T	T ←
T	T	F	T	T	$\cancel{F}$ ←
T	F	T	F	F	T
T	F	F	F	T	F
F	T	T	T	T	T ←
F	T	F	T	T	T ←
F	F	T	T	F	T
F	F	F	T	T	T ←

Modus Ponens

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Modus Tollens

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

Generalization

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

Specialization

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

Elimination

$$\begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$