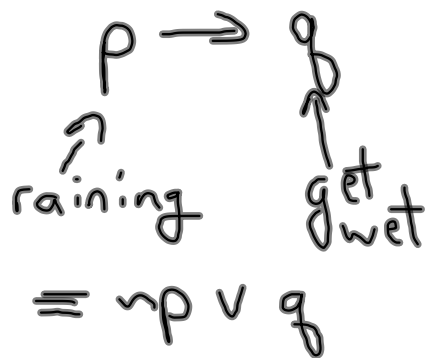


$$p \rightarrow q \equiv \sim p \vee q$$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

negation of conditional
 $p \wedge \sim q$



$\sim q \rightarrow \sim p$
 contrapositive

$\equiv \sim(\sim q) \vee \sim p$
 $\equiv q \vee \sim p$

p	q	$\sim p$	
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

converse of $p \rightarrow q$

$$q \rightarrow p$$

$\sim q \vee p$

p	q	$\sim q$	\rightarrow
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

inverse

$$\neg p \rightarrow \neg q \equiv q \rightarrow p$$

p only if q

$$\neg q \rightarrow \neg p$$

$$p \rightarrow q$$

Biconditional

• p if and only if q

p if q $q \rightarrow p$
 p only if q $p \rightarrow q$

• $p \leftrightarrow q$ $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$
• p iff q

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

p : dinner
 q : dessert

$$(p \rightarrow q) \wedge \begin{matrix} \cancel{(q \rightarrow p)} \\ (\sim p \rightarrow \sim q) \end{matrix}$$

arguments

If you eat dinner then
you can have dessert.

You ate dinner.

Therefore You can have dessert

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

~~$$\begin{array}{l} p \rightarrow q \\ q \\ \hline \therefore p \end{array}$$~~

$$\frac{P \rightarrow q}{P} \quad \left. \vphantom{\frac{P \rightarrow q}{P}} \right\} \text{premise}$$

$$\therefore q \quad \left. \vphantom{\therefore q} \right\} \text{conclusion}$$

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

valid: whenever all premises are true, the conclusion is also true

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

not valid

$P \wedge Q$
 $\therefore P$
 valid

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

$P \vee Q$
 $\therefore P$
 not valid

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

$\begin{array}{c|c} p & r \\ \hline p & \rightarrow \\ r & \rightarrow \end{array}$

	p	$\neg p$		$p \rightarrow q$	$r \rightarrow q$	$p \rightarrow r$
p	\neg	\neg	\neg	\neg	\neg	\neg
$\neg p$	\neg	\neg	\neg	\neg	\neg	\neg
p	\neg	\neg	\neg	\neg	\neg	\neg
$\neg p$	\neg	\neg	\neg	\neg	\neg	\neg

Modus Ponens

$$\frac{p \rightarrow q}{p} \\ \therefore q$$

Modus Tollens

$$\frac{p \rightarrow q}{\sim q} \\ \therefore \sim p$$

Generalization

$$\frac{p}{\therefore p \vee q}$$

Specialization

$$\frac{p \wedge q}{\therefore p}$$

Elimination

$$\frac{p \vee q}{\sim q} \\ \therefore p$$