25)

$\forall a, b \in \mathbb{Z}$ if $a \bmod 7 = 5$ and $b \bmod 7 = 6$ then $ab \bmod 7 = 2$

---

Suppose $a, b \in \mathbb{Z}$ and $a \bmod 7 = 5$ and $b \bmod 7 = 6$

Show $ab \bmod 7 = 2$

---

$a \bmod 7 = 5$

so $a = 7q + 5 \quad q \in \mathbb{Z}$

and $b \bmod 7 = 6$

so $b = 7r + 6 \quad r \in \mathbb{Z}$

now, $ab = (7q + 5)(7r + 6)$

$= 49qr + 42q + 35r + 30$

$= 49qr + 42q + 35r + 28 + 2$

$= 7(7qr + 6q + 5r + 4) + 2$

Let $k = (7qr + 6q + 5r + 4)$

so $k \in \mathbb{Z}$

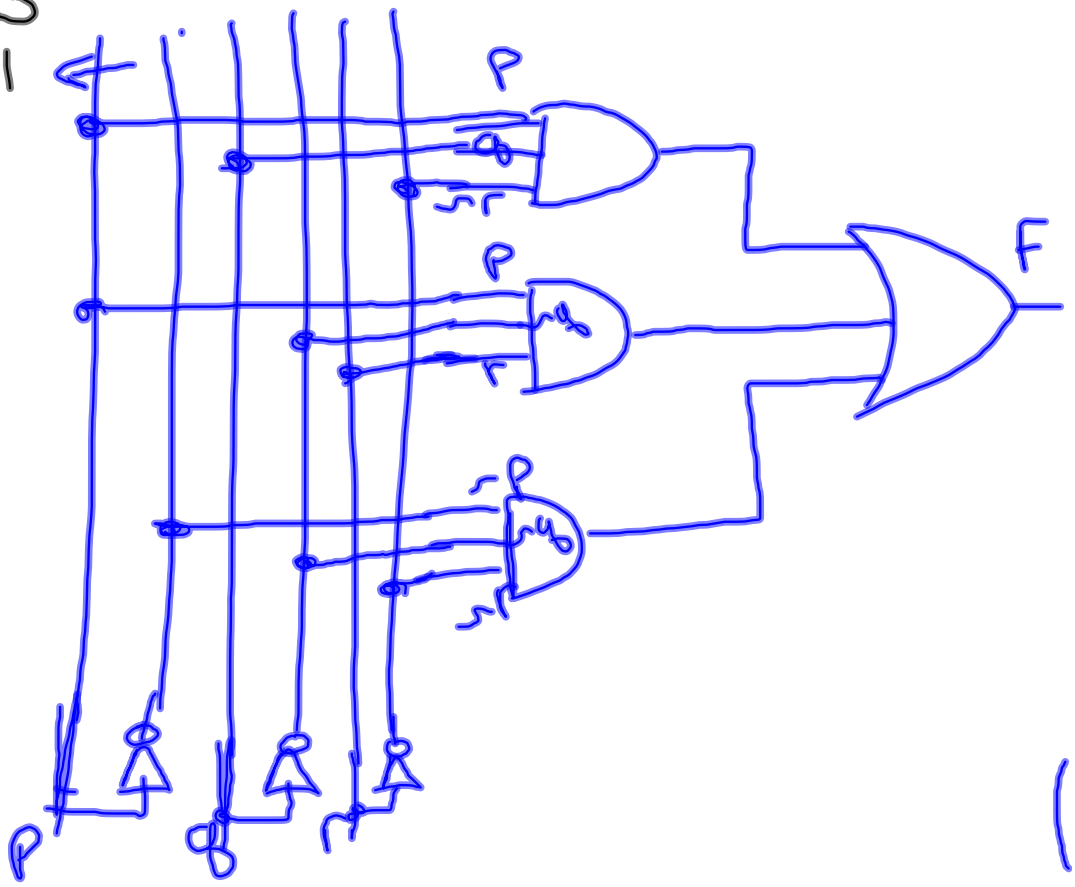
and $ab = 7k + 2$

$ab = 7k + 2, \; k \in \mathbb{Z}$

$\therefore ab \bmod 7 = 2$ QED

| p | q | r | f |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

$$F(p,q,r) = (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$



if you miss the test, then you will fail it.

$p \to q$

$\equiv \neg p \vee q$

---

negation:

$p \wedge \neg q$

You missed the test and you did not fail it.

4.5 #24

$\forall m \in \mathbb{Z}, x \in \mathbb{R}$, if $x$ is not an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$

Suppose $m \in \mathbb{Z}, x \in \mathbb{R}, x \notin \mathbb{Z}$

so $x = r + f$ where $r \in \mathbb{Z}$ and $0 < f < 1$

now $\lfloor x \rfloor = r$

$\lfloor m - x \rfloor = m - r - 1$

$\lfloor x \rfloor + \lfloor m - x \rfloor = r + m - r - 1$

$= m - 1$