2.3 #40

who killed?

S	F	L	M	S	F	L	M	
T	F	F	F			✓	✓	X
F	T	F	F					
F	F	T	F					
F	F	F	T			✗	✓	

The sum of two odd integers is even.

Proof:

Suppose m and n are odd integers. By the definition of odd, $m = 2r+1$ and $n = 2s+1$ for some integers r and s.

Then

$$m+n = 2r+1+2s+1 \quad \text{by substitution}$$
$$= 2r+2s+2$$
$$= 2(r+s+1)$$

Let $k = r+s+1$. Note that k is an integer because it is the sum of integers. Hence, $m+n = 2k$ where k is an integer.

It follows from the definition of even that $m+n$ is even.

QED

writing proofs
common mistakes

lists in your book

r is rational $\Leftrightarrow$

$\exists$ integers a, b s.t. $r = \frac{a}{b}$

and $b \neq 0$.

The sum of any two rationals is rational.

suppose

$\forall r, s$, if r and s are rational

then r+s is rational.

show

Suppose r and s are rational
(show r+s is rational)

So $r = \frac{a}{b}$ and $s = \frac{c}{d}$ where

$b \neq 0$ and $d \neq 0$

consider

$$r + s = \frac{a}{b} + \frac{c}{d} \quad \text{subst.}$$

$$= \frac{ad + cb}{b \cdot d}$$

Let $e = ad + cb$ and $f = b \cdot d$

so $r + s = \frac{e}{f}$ and e, f are integers since they are the sum and product of integers.

so $f \neq 0$ by zero product property (since $b \neq 0$ and $d \neq 0$)

$r + s = \frac{e}{f}$ and $f \neq 0$, $e, f \in \mathbb{Z}$

$\therefore$ r+s is rational. QED

$\forall m, n \in \mathbb{Z}$, if m is even and n is odd then $\underline{m^2+3n}$ is odd.

m^2 is even (prod. of 2 evens is even)

$3n$ is odd (prod of 2 odds is odd)

m^2+3n is odd (even + odd is odd)

QED

Divisibility

If n and d are integers
then

n is divisible by d iff
$n = dk$ for some $k \in \mathbb{Z}$

n is a multiple of d
d is a factor of n
d is a divisor of n
d divides n
$d \mid n$

if $n, d \in \mathbb{Z}$ and $d \neq 0$
$d \mid n \Leftrightarrow \exists\, k \in \mathbb{Z}$ s.t. $n = dk$

$3 \mid 27$
$2 \nmid 15$