$$\sim\left(\forall x\in\mathbb{R},\ \exists y\in\mathbb{R} \text{ s.t. } x\cdot y=1 \text{ or } x+y=0\right)$$

$$\exists x\in\mathbb{R} \text{ s.t. } \sim(\exists y \dots\quad)$$

$$\exists x\in\mathbb{R} \text{ s.t. } \forall y\in\mathbb{R}, \sim(\qquad)$$

$$\text{''} \quad \text{''} \quad \text{''} \ , \ x\cdot y\neq 1 \text{ and } x+y\neq 0)$$

$\forall x$ if $P(x)$ Then $Q(x)$

$P(a)$ for a particular a

$\therefore Q(a)$

Universal modus tollens

$\forall x, P(x) \rightarrow Q(x)$

$\neg Q(a)$

$\therefore \neg P(a)$

if and only if

$n \in \mathbb{Z}$ is even $\iff$

$\exists k \in \mathbb{Z}$ s.t. $n = 2 \cdot k$

$n \in \mathbb{Z}$ is odd $\iff$
$\exists k \in \mathbb{Z}$ s.t $n = 2k+1$

$\boxed{n \in \mathbb{Z}}$ is prime $\iff \forall r, s \in \mathbb{Z}^+$,
for $n > 1$ if $n = r \cdot s$ then
$r = 1$ or $s = 1$

n is composite $\iff \exists r, s \in \mathbb{Z}^+$ s.t.
$n = r \cdot s$ and $r \neq 1$ and $s \neq 1$

Proving Existential Statements

$$\exists x \in D \text{ s.t. } P(x)$$

provide directions for determining x, then show $P(x)$ is true.

Construction Proof

There is an even integer that is the sum of two prime numbers.

Let $n = 4$

$4 = 2 + 2$ and 2 is prime

Suppose r and s are integers
Prove that there is an integer
k s.t. $22r + 18s = 2 \cdot k$.

Let $k = 11r + 9s$
k is an integer ($\mathbb{Z}$ closed under $+, \times$)
$2 \cdot k = 2 \cdot (11r + 9s)$ substitution
$= 22 \cdot r + 18s$ algebra