$$\forall x \in D,\ P(x)$$

$$\exists x \in D \text{ s.t. } P(x)$$

$$\sim(\forall x \in D,\ P(x))$$

$$\equiv \exists x \in D \text{ s.t. } \sim P(x)$$

$$\sim(\exists x \in D \text{ s.t. } P(x))$$

$$\equiv \forall x \in D,\ \sim P(x)$$

$$\forall x \in D,\ P(x) \to Q(x)$$

$$\sim\left(\forall x \in D,\ P(x) \to Q(x)\right)$$

$$\equiv \exists x \in D \text{ s.t. } \sim\left(P(x) \to Q(x)\right)$$

$$\equiv \exists x \in D \text{ s.t. } \sim\left(\sim P(x) \vee Q(x)\right)$$

$$\equiv \exists x \in D \text{ s.t. } P(x) \wedge \sim Q(x)$$

All lizards in this classroom have six legs.

$$\forall \ell \in \text{Lizards}, \ \text{Classroom}(\ell) \rightarrow \text{SixLegs}(\ell)$$

$$\forall x \in \text{Students},\ GW(x) \rightarrow Degree(x)$$

All students who do great work get a degree.

contrapositive:

$$\forall x \in \text{Students},\ \sim Degree(x) \rightarrow \sim GW(x)$$

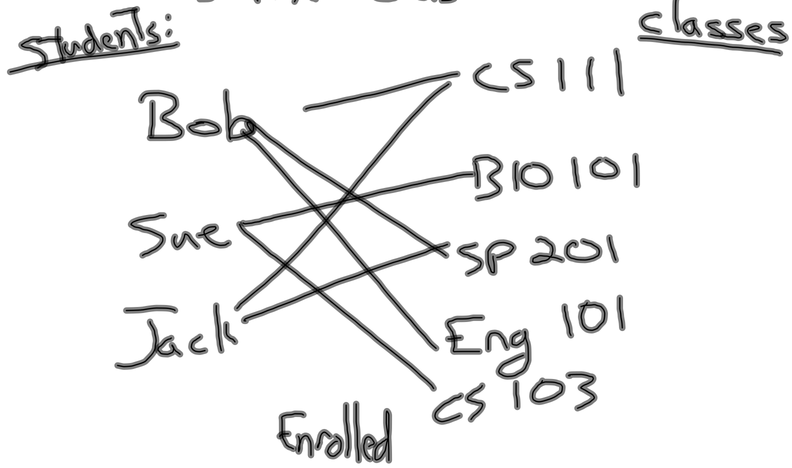
converse

$$\forall x \in \text{Students},\ Degree(x) \rightarrow GW(x)$$

inverse

$$\forall x \in \text{Students},\ \sim GW(x) \rightarrow \sim Degree(x)$$

1. All students are enrolled in some class.



$\forall s \in \text{Students}, \exists c \in \text{Class} \text{ s.t. } \text{Enrolled}(s,c)$

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2. There is a class taken by every student.

$\exists c \in \text{Class} \text{ s.t. } \forall s \in \text{Students}, \text{Enrolled}(s,c)$

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negation of 1

$\sim\left(\forall s \in \text{students}, \exists c \in \text{Class} \text{ s.t } \text{Enrolled}(s,c)\right)$

$\equiv \exists s \in \text{students} \text{ s.t. } \sim\left(\exists c \in \text{Class s.t. } \text{Enrolled}(s,c)\right)$

$\equiv \exists s \in \text{Students} \text{ s.t. } \forall c \in \text{Classes}, \sim \text{Enrolled}(s,c)$

There is a smallest positive integer.

$$\exists\, m \in \mathbb{Z}^+ \text{ s.t. } \forall n \in \mathbb{Z}^+,\ m \le n$$

$$\cancel{\mathbb{Z}^+ - \{m\}}$$

Arguments

All computers have memory
gbcs1 is a computer
$\therefore$ gbcs1 has memory

Universal Instantiation

$$\forall x \in D, P(x)$$
$$gbcs1 \in D$$
$$\therefore P(gbcs1)$$

Universal Modus Ponens

If a number is even, then its square is even.

4 is an even number

$\therefore$ 4^2 is even

$\forall x \in D,\ P(x) \rightarrow Q(x)$

$P(a)$ for a particular a

$\therefore$ $Q(a)$