

Predicate Calculus

Predicate: a sentence containing variables and becomes a statement when specific values are assigned to the variables.

pred. $\rightarrow P(x): x < 5$

statements $\rightarrow P(3)$ true
 $\rightarrow P(7)$ false

variable has a Domain

Truth set of $P(x)$ — From the domain
- all x values that make predicate P true.

$\{x \in D \mid P(x)\}$
set of all values, x , from the domain, D such that $P(x)$ is true.

$Q(n)$: n is a factor of 8

Truth set	domain
$\{1, 2, 4, 8\}$	$\mathbb{Z}^+$
$\{-1, -2, -4, -8, 1, 2, 4, 8\}$	$\mathbb{Z}$

Universal Quantifier

"for all", "every"

$\forall$

statement

$\forall x \in H, x \text{ is mortal}$

H : humans

predicate

$$\forall x, y \in \mathbb{R}, \underbrace{x + y = y + x}_{(x, y)}$$

a universal statement, $\forall x \in D, Q(x)$
is true iff $Q(x)$ is true for
every x in D

if and only if

$$p \rightarrow q$$

$$q \rightarrow p$$

it is false iff $Q(x)$ is
false for at least one $x \in D$

a value for $x \in D$ for which
 $Q(x)$ is false is called
a counterexample.

$$P(x): x^2 \geq x$$

$$\forall x \in D, x^2 \geq x$$

$$D = \{1, 2, 3, 4, 5\}$$

verify:
check every value
exhaustion

$$\forall x \in \mathbb{R}, x^2 \geq x$$

$$\text{suppose } x = \frac{1}{2}$$

$$x^2 = \frac{1}{4}$$

so statement is false

Existential Quantifier

There exists

There is

for some

$\exists$

$\exists s \in S$ such that s is a student
 S : set of all people in this room

$$\exists m \in \mathbb{Z} \text{ s.t. } m^2 = m$$

for $m=1$, $m^2=1$ so true

$$D = \{5, 6, 7, 8, 9\}$$

$$\exists m \in D \text{ s.t. } m^2 = m$$

false

$$P(x) \Rightarrow Q(x)$$

$$\forall x, P(x) \rightarrow Q(x)$$

every element in the truth set of $P(x)$ is also in the truth set of $Q(x)$

$$P(x) \iff Q(x)$$

$$\forall x, P(x) \leftrightarrow Q(x)$$

$P(x), Q(x)$ have identical truth sets

All Java programs have at least 5 lines.

$P(x)$: x is a Java program

$Q(x)$: x has at least 5 lines

$$\forall x, P(x) \rightarrow Q(x)$$

$$x \in \text{files}$$

$$\forall x \in P(x), Q(x)$$