$$b_k = 4b_{k-1} + 5 \qquad k \geq 1 \qquad b_0 = 2$$

$$b_0 = 2$$

$$b_1 = 4 \cdot 2 + 5$$

$$b_2 = 4(4 \cdot 2 + 5) + 5 = 4^2 \cdot 2 + 4 \cdot 5 + 5$$

$$b_3 = 4(4(4 \cdot 2 + 5) + 5) + 5$$

$$= 4^3 \cdot 2 + 4^2 \cdot 5 + 4 \cdot 5 + 5$$

$$b_4 = 4(\qquad) = 4^4 \cdot 2 + 4^3 \cdot 5 + 4^2 \cdot 5 + 4 \cdot 5 + 5$$

$$b_n = 4^n \cdot 2 + \sum_{i=0}^{n-1} 5 \cdot 4^i$$

$$\boxed{\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}}$$

$$= 4^n \cdot 2 + 5 \cdot \sum_{i=0}^{n-1} 4^i$$

$$= 4^n \cdot 2 + 5 \cdot \left(\frac{4^n - 1}{\underbrace{4 - 1}_{3}} \right)$$

$a_1 = 1$, $a_2 = 3$

$a_k = a_{k-2} + 2 \cdot a_{k-1}$ for $k \geq 3$

Prove a_n is odd for all $n \geq 1$, $n \in \mathbb{Z}$

Basis: $a_1 = 1$ which is odd

$a_2 = 3$ " " "

Induction

Suppose ~~$a_k = a_{k-2} + 2 \cdot a_{k-1}$~~

$a_i = a_{i-2} + 2 \cdot a_{i-1}$

$\forall i, \ 1 \leq i < k$

a_i is odd

Show $a_k = a_{k-2} + 2 \cdot a_{k-1}$ is odd

consider $a_k = a_{k-2} + 2 \cdot a_{k-1}$

a_{k-2} is odd ← ind hyp

a_{k-1} is odd so $2 \cdot a_{k-1}$ is even

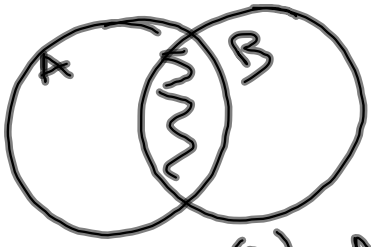
and an even plus an odd is odd

Multplication

Addition Rule

two <u>disjoint</u> sets

A+B



$N(A) + N(B) - N(A \cap B)$