\[ b_k = 4b_{k-1} + 5 \quad k \geq 1 \quad b_3 = 2 \]

\[ b_0 = 2 \]
\[ b_1 = 4 \cdot 2 + 5 \]
\[ b_2 = 4(4 \cdot 2 + 5) + 5 = 4^2 \cdot 2 + 4 \cdot 5 + 5 \]
\[ b_3 = 4(4(4 \cdot 2 + 5) + 5) + 5 \]
\[ = 4^3 \cdot 2 + 4^2 \cdot 5 + 4 \cdot 5 + 5 \]
\[ b_4 = 4(4^3 \cdot 2 + 4^2 \cdot 5 + 4 \cdot 5 + 5) + 5 \]
\[ b_n = 4^n \cdot 2 + \sum_{i=0}^{n-1} 5 \cdot 4^i \]
\[ = 4^n \cdot 2 + 5 \cdot \sum_{i=0}^{n-1} 4^i \]
\[ = 4^n \cdot 2 + 5 \cdot \left( \frac{4^n - 1}{4 - 1} \right) \]
\( a_1 = 1, \quad a_2 = 3 \)

\( a_k = a_{k-2} + 2 \cdot a_{k-1} \quad \text{for} \quad k \geq 3 \)

Prove \( a_n \) is odd for all \( n \geq 1 \)

Basis: \( a_1 = 1 \) which is odd
\( a_2 = 3 \) " " "

Induction

Suppose \( a_k = a_{k-2} + 2 \cdot a_{k-1} \)

\[ a_i = a_{i-2} + 2 \cdot a_{i-1} \]

\( \forall i \), \( 1 \leq i < k \)

\( a_i \) is odd

Show \( a_k = a_{k-2} + 2 \cdot a_{k-1} \)

is odd

Consider \( a_k = a_{k-2} + 2 \cdot a_{k-1} \)

\( a_{k-2} \) is odd \( \text{ind hYP} \)

\( a_{k-1} \) is odd so \( 2 \cdot a_{k-1} \)

is even

and an even plus an odd is odd
Multiplication

Addition Rule

two disjoint sets

\[ A + B = N(A) + N(B) - N(A \cap B) \]