Friday: Review

Quiz 1: Bring text

Quiz 2: 1 exam question/answer
-email / dropbox

$$\omega_1 = 1$$
$$\omega_k = 1 + \omega_{\lfloor k/2 \rfloor}$$

$$\omega_1 = 1$$
$$\omega_2 = 1 + \omega\lfloor 2/2 \rfloor = 1 + \omega_1 = 1 + 1 = 2$$
$$\omega_3 = 1 + \omega\lfloor 3/2 \rfloor = 2$$
$$\omega_4 = 1 + \omega\lfloor 4/2 \rfloor = 1 + \omega_2 = 3$$
$$w_5 = 1 + w\lfloor 5/2 \rfloor = 1 + w_2 = 3$$
$$w_6 = 1 + w_{\lfloor 6/2 \rfloor} = 1 + w_3 = 3$$
$$w_7 = 1 + w\lfloor 7/2 \rfloor = 1 + w_3 = 3$$
$$w_8 = 1 + w_{\lfloor 8/2 \rfloor} = 1 + w_4 = 4$$
$$\vdots$$
$$w_{15} = 1 + w_{\lfloor 15/2 \rfloor} = 1 + w_7 = 4$$
$$w_{16} = 1 + w_{\lfloor 16/2 \rfloor} = 1 + w_8 = 5$$
$$\vdots$$

$2^2 \le n < 2^3$ $\quad$ $2^3 \le n < 2^4$

$\omega_n = \cancel{2}3$ $\quad$ $w_n = \cancel{3}4$

$$\boxed{\begin{array}{c} 2^i \le n < 2^{i+1} \\ w_n = i+1 \end{array}}$$

$$\boxed{w_n = \lfloor \log_2 n \rfloor + 1}$$

$k \in \mathbb{Z}$, $x \in \mathbb{R}$ w/

$$2^k \le x < 2^{k+1}$$

$$\lfloor \log_2 x \rfloor = k$$

if $w_1 = 1$

$w_n = 1 + w_{\lfloor n/2 \rfloor}$

then

$w_n = \lfloor \log_2 n \rfloor + 1$ for $n \geq 1$

Suppose $w_i = \lfloor \log_2 i \rfloor + 1$ for $1 \leq i < k$

Show $w_k = \lfloor \log_2 k \rfloor + 1$

case 1: k is odd $\quad \lfloor \frac{k}{2} \rfloor = \frac{k-1}{2}$

$$w_k = 1 + w_{\lfloor k/2 \rfloor}$$

$$= 1 + w_{\frac{k-1}{2}} \qquad \frac{k-1}{2} < k$$

$$= 1 + \left(\left\lfloor \log_2 \left(\frac{k-1}{2} \right) \right\rfloor + 1 \right) \quad \text{from ind. hyp.}$$

$$= 2 + \lfloor \log_2(k-1) - \log_2(2) \rfloor$$

$$= 2 + \lfloor \log_2(k-1) - 1 \rfloor$$

$$= 2 + \lfloor \log_2(k-1) \rfloor - 1$$

$$= \lfloor \log_2 (k-1) \rfloor + 1$$

since k is odd, $k > 1$

$$\lfloor \log_2(k-1) \rfloor = \lfloor \log_2(k) \rfloor$$

$$= \lfloor \log_2(k) \rfloor + 1$$

$\therefore$ even case is similar

$w_n = \lfloor \log_2 n \rfloor + 1$

$w_n = \lfloor \log_2 n + 1 \rfloor$

$\log_2 n \quad < w_n \leq \log_2 n + 1$

$x = \lfloor y \rfloor$

then

$y - 1 < x \leq y$

$w_n \leq \log_2 n + 1$

$\leq \log_2 n + \log_2 n$

$2 \cdot \log_2 n$

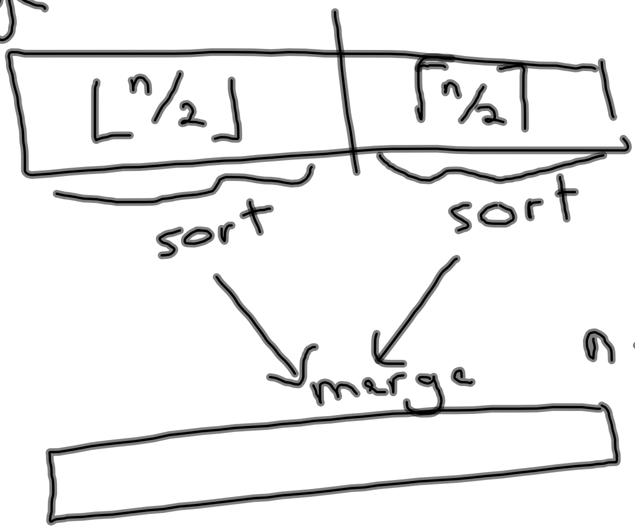
$1 \leq \log_2 n$ for $n \geq 2$

$A \cdot \log_2 n < w_n \leq B \cdot \log_2 n$ for all $n \geq k$

$A = 1$ $\quad B = 2$ $\quad k = 2$

so w_n is $\Theta(\log_2 n)$

Merge Sort



$n-1$ comparisons