

```

int getFirst (int[] data){
    if (data.length < 1)
        throw new NoSuchElementException();
    return data[0];
}

```

---

length (n)

10

100

1000 000

array accesses

1

1

1

$O(1)$

constant

$p = 0$

$x = 2$

for (int  $i = 2$ ;  $i \leq n$ ;  $i++$ ) {

$p = (p+1) * x$ ;

}

iterations of the loop  $n - 2 + 1 = n - 1$

---

additions: control loop:  $n - 1$   
in the loop:  $n - 1$

multiplications:  $n - 1$

$3(n - 1)$

$O(n)$

$\Omega(n)$

$\Theta(n)$

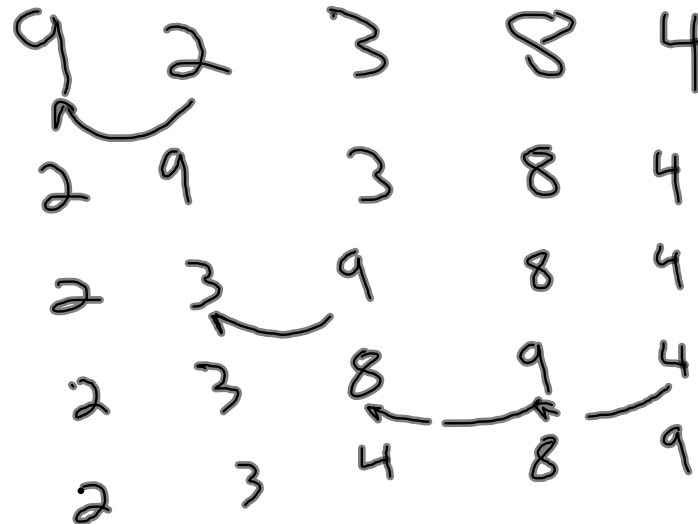
```

      s=0
      for (int i=1; i≤n; i++) {
          for (int j=1; j≤i; j++) {
              S = S + j · (i - j + 1);
          }
      }
  
```

$5 \cdot i$

$$\begin{aligned}
 n + \sum_{i=1}^n 5i &= n + 5 \cdot \sum_{i=1}^n i \\
 &= n + 5 \left( \frac{n(n+1)}{2} \right) \\
 &= n + 5 \left( \frac{1}{2}n^2 + \frac{1}{2}n \right) \\
 &= n + \frac{5}{2}n^2 + \frac{5}{2}n = \frac{3}{2}n + \frac{5}{2}n^2 \\
 &O(n^2)
 \end{aligned}$$

# Insertion Sort



$n$ 
 $\left[ \begin{array}{l} \text{for each item } i \text{ at index } \text{idx} \\ \text{move } i \text{ to the left as long} \\ \text{as it is smaller than the one} \\ \text{before} \end{array} \right.$

$$\Theta(n^2)$$

$$y = 2^x$$

$$x = \log_2 y$$

Binary Search

$a[1]..a[n]$   
search for  $x$

index = 0  
low = 1  
high = n

while (high  $\geq$  low && index = 0) {

mid =  $\lfloor \frac{high+low}{2} \rfloor$

if ( $a[mid] == x$ )

index = mid

if ( $a[mid] > x$ )

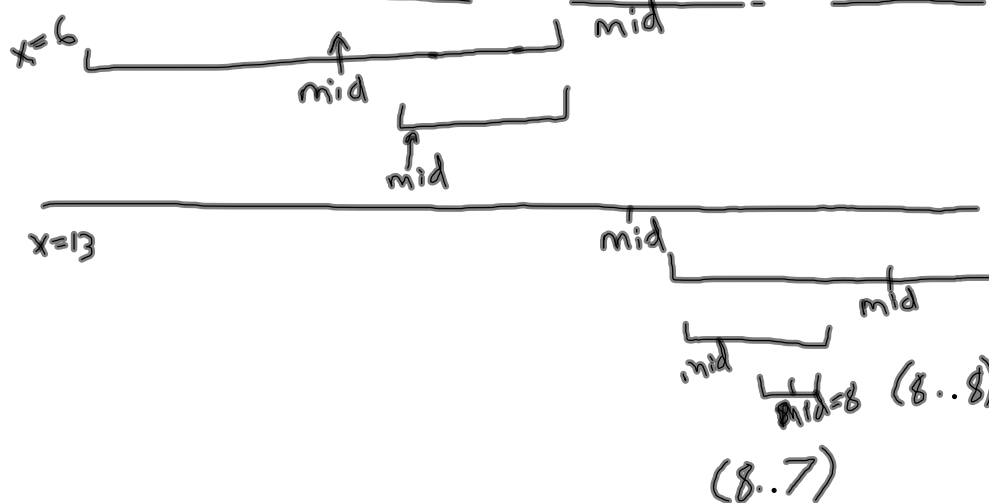
high = mid - 1

else

low = mid + 1

}

|    |   |   |   |   |   |   |    |    |    |    |    |
|----|---|---|---|---|---|---|----|----|----|----|----|
| i  | 1 | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 |
| a: | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 15 | 18 | 21 | 2  |



start w/  $k$  elements  $1 \dots k$

$$\text{mid} = \left\lfloor \frac{k+1}{2} \right\rfloor$$

$$1 \dots \left\lfloor \frac{k+1}{2} \right\rfloor - 1 \quad \text{or} \quad \left\lfloor \frac{k+1}{2} \right\rfloor + 1 \dots k$$

$k$  is even      mid  $\frac{k}{2}$

$$\begin{array}{c} \boxed{1 \ 2 \ 3 \ 4} \\ \uparrow \\ \text{mid} \end{array}$$

$$\underbrace{1 \dots \frac{k}{2} - 1}_{\left(\frac{k}{2} - 1\right) - 1 + 1 = \frac{k}{2} - 1}, \quad \frac{k}{2}, \quad \underbrace{\frac{k}{2} + 1 \dots k}_{k - \left(\frac{k}{2} + 1\right) + 1 = k - \frac{k}{2} - 1 + 1 = \frac{k}{2}}$$

$k$  is odd      mid  $\frac{k+1}{2}$

$$\underbrace{1 \dots \frac{k+1}{2} - 1}_{\left\lfloor \frac{k}{2} \right\rfloor}, \quad \frac{k+1}{2}, \quad \underbrace{\frac{k+1}{2} + 1 \dots k}_{\left\lfloor \frac{k}{2} \right\rfloor}$$

$$\omega_k = 1 + \omega_{\left\lfloor \frac{k}{2} \right\rfloor}$$

$$\omega_1 = 1$$