

Programs

- finite length string
of 0's and 1's

set of all programs

n	F(n)
1	0
2	1
3	00
4	01
5	10
6	11
7	000
⋮	⋮

countable

Functions $G: \mathbb{Z}^+ \rightarrow \{0, 1, 2, \dots, 9\}$

T set of all such functions

T is uncountable

Let $S = \mathbb{R}$ between $0..1$

Define $f: S \rightarrow$ ~~T~~ subset of T

$f(0.a_1a_2a_3 \dots a_n \dots) =$ function

$$f(n) = a_n$$

subset of T for co-domain
is the image (range) of f

so f is onto

$$f(x_1) = f(x_2)$$

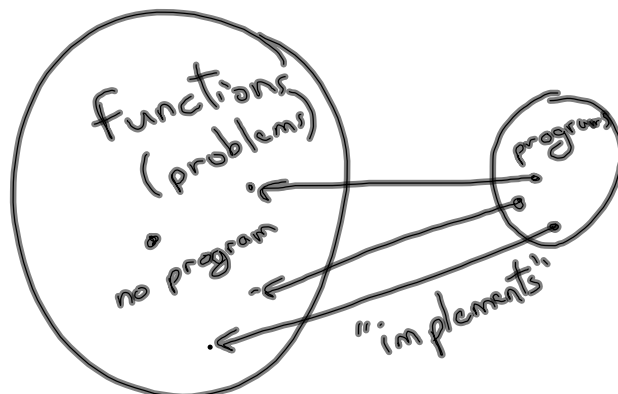
$$\hookrightarrow f^1$$

$$\hookrightarrow f^2$$

so $x_1 = x_2 \rightarrow$ so f
is one to one

so subset of T is uncountable

so T is "



for all integers $n \geq 0$,
 $2^n < (n+2)!$

Basis: $n=0$

$$2^0 = 1$$

$$(0+2)! = 2! = 2$$

$$0 < 2 \checkmark$$

Induction:
suppose statement is true
for $n=k$
 $2^k < (k+2)!$

[show true for $n=k+1$ so
 $2^{k+1} < ((k+1)+2)!$]

$$2^{k+1} = 2 \cdot 2^k < 2 \cdot (k+2)!$$

$$< 3 \cdot (k+2)!$$

$$< (3+k) \cdot (k+2)!$$

$$= (k+3)(k+2)!$$

$$= ((k+1)+2)!$$

subst ind
hyp

$$X = \{a, b\}$$

$$Y = \{u, v\}$$

$$X \times Y = \{(a, u), (a, v), (b, u), (b, v)\}$$

$$\mathcal{P}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$\begin{aligned} \mathcal{P}(X \times Y) = & \{\emptyset, \{(a, u)\}, \{(a, v)\}, \dots \\ & \{(a, u), (a, v)\}, \dots \\ & \{(b, u), (b, v)\}, \dots \\ & \{(a, u), (a, v), (b, u), (b, v)\}\} \end{aligned}$$