

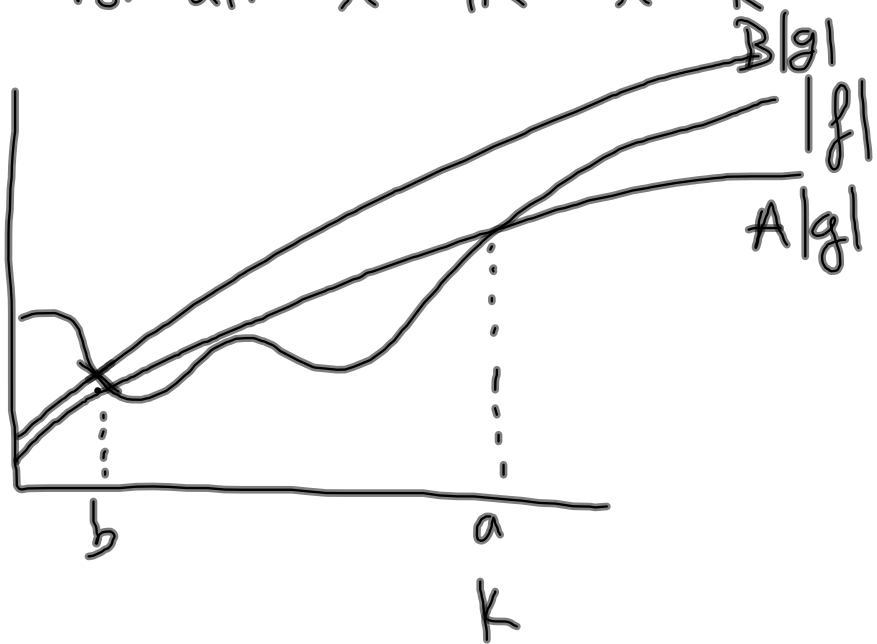
(H)

$f(x)$ is (H)($g(x)$) iff

$\exists A, B, k \in \mathbb{Z}^+$ s.t.

$$A|g(x)| \leq |f(x)| \leq B|g(x)|$$

for all $x \in \mathbb{R}$ $x > k$



- $f(x)$ is $\Theta(g(x))$ iff
 $f(x)$ is $\Omega(g(x))$ and
 $f(x)$ is $O(g(x))$

- $f(x)$ is $\Omega(g(x))$ iff
 $g(x)$ is $O(f(x))$

Show $2x^4 + 3x^3 + 5$ is $\Theta(x^4)$

$$f(x) = 2x^4 + 3x^3 + 5$$

$$g(x) = x^4$$

I. show $f(x)$ is $\Omega(g(x))$

II. show $f(x)$ is $O(g(x))$

I find A, a such that $|f(x)| \geq A|g(x)|$
for all $x > a$

$$f(x) = 2x^4 + \underbrace{3x^3 + 5}$$

$$3x^3 + 5 > 0 \text{ for } x > 0$$

$$\text{so } 2x^4 + 3x^3 + 5 > 2x^4 \text{ for } x > 0$$

since all terms are positive

$$|2x^4 + 3x^3 + 5| > 2|x^4| \text{ for } x > 0$$

by def of Ω w/ $A=2$ and $a=0$,
 $f(x)$ is $\Omega(g(x))$

II. show $f(x)$ is $O(g(x))$

$$f(x) = 2x^4 + 3x^3 + 5$$

$$\text{so } f(x) \leq 2x^4 + 3x^4 + 5x^4 \text{ for } x > 1$$



$$10x^4$$

$$\text{so } f(x) \leq 10x^4 \text{ for } x > 1$$

$\uparrow \quad \quad \quad \uparrow$
 $B \quad \quad \quad b$

$$|2x^4 + 3x^3 + 5| \leq 10|x^4|$$

\therefore by def of O w/ $B=10$
and $b=1$, $f(x)$ is $O(g(x))$

since $f(x)$ is $O(g(x))$ and
 $f(x)$ is $\Omega(g(x))$ we conclude
that $f(x)$ is $\Theta(g(x))$

QED

for $a_0, a_1, a_2, \dots, a_n \in \mathbb{R}$
and $a_n \neq 0$

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

is $\mathcal{O}(x^s) \quad \forall s \geq n$

$\Omega(x^r) \quad \forall r \leq n$
 $\Theta(x^n)$

is NOT $\mathcal{O}(x^m) \quad m < n$

$\Theta(x^p) \quad p \neq n$

$$m, n \in \mathbb{Z}^+ \quad r_i, s_i \in \mathbb{Q}^+$$

$$\text{s.t. } r_0 < r_1 < r_2 < r_3 \dots < r_n$$

$$s_0 < s_1 < s_2 < \dots < s_m$$

$$a_j, b_j \in \mathbb{R} \quad a_n \neq 0$$

$$b_m \neq 0$$

$$\frac{a_n x^{r_n} + a_{n-1} x^{r_{n-1}} + \dots + a_1 x^{r_1} + a_0 x^{r_0}}{b_m x^{s_m} + b_{m-1} x^{s_{m-1}} + \dots + b_1 x^{s_1} + b_0 x^{s_0}}$$

$$\text{is } \mathbb{H} \left(x^{r_n - s_m} \right)$$

$$\text{eg. } \frac{6x^2 + 2x + 3}{9x^4 + 3x^3 + x} \text{ is } \mathbb{H} \left(x^{-2} \right)$$

Efficiency of Algorithms

- time (runtime)

- space (memory)

factors influencing time

- algorithm

- nature of the data

- size of data (n)

cases:

best

worst

average

1

n

$n/2$

} # of operations

operations to count

- "instructions"

- array accesses

↳ memory accesses

- add/subtract

mul/div

- disk accesses

- assignments