Series

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{k=0}^{3} 2^k = 2^0 + 2^1 + 2^2 + 2^3$$

$$= 1 + 2 + 4 + 8 = 15$$

$$\sum_{k=m}^{n} a_k = \left(\sum_{k=m}^{n-1} a_k\right) + a_n$$

$$= a_m + \left(\sum_{k=m+1}^{n} a_k\right)$$

$$\frac{1}{k} - \frac{1}{k+1} = \frac{(k+1)-(k)}{k(k+1)} = \frac{1}{k(k+1)}$$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \left(\frac{1}{1} \cancel{- \frac{1}{2}}\right) \cancel{+ \left(\frac{1}{2}\right.} \cancel{- \frac{1}{3}}\right) \cancel{+ \left(\frac{1}{3}\right.} \cancel{- \frac{1}{4}}\right)$$

$$+ \cdots \cancel{+ \left(\frac{1}{n}\right.} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

Products

$$\prod_{k=1}^{n} a_k = a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$$

$$= a_1 \cdot \prod_{k=2}^{n} a_k$$

$$= \prod_{k=1}^{n-1} a_k \cdot a_n$$

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k$$

$$\sum_{k=1}^{4} 2^{(k-1)} = 2^0 + 2^1 + 2^2 + 2^3 = \sum_{j=0}^{3} 2^j$$

Factorial

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$$

$$= \prod_{i=1}^{n} i$$

$$0! = 1$$

Induction

$P(n)$: a property that we want to prove.
(for $n \geq a$)

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1. $P(a)$ is true
2. for all $k \geq a$, if $P(k)$ than it is true for $P(k+1)$

Conclude that
for all integers $n \geq a$, $P(n)$

Statement: $\forall\, n \geq a,\ n, a \in \mathbb{Z},\ P(n)$

Proof:

Step 1: (Basis) Show $P(a)$ is true.

Step 2: (Induction) Show for all $k \geq a$ if $P(k)$ then $P(k+1)$

Suppose $P(k)$

Show $P(k+1)$

Conclude $P(n)$ for $n \geq a$

for $n \geq 1$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Proof (by Mathematical Induction)

Basis: Show true for $n=1$

$\sum_{i=1}^{1} i = 1$

$\frac{n(n+1)}{2} = \frac{1(2)}{2} = 1$

so $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for $n=1$

Induction:

Suppose it is true for $n=k$ for some integer $k \geq 1$

[Show it is true for $n=k+1$]

Suppose $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ ← induction hypothesis

[Show $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$]

Consider $\sum_{i=1}^{k+1} i = \left(\sum_{i=1}^{k} i\right) + (k+1)$ sep. last term

$= \left(\frac{k(k+1)}{2}\right) + (k+1)$

by substitution w/ the induction hyp.

$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$= \frac{k(k+1) + 2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$ factor out $(k+1)$