Algorithms

~~set~~ list of steps for completing a task.

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assignment

$x := 5;$ $\quad x \leftarrow 5;$

Division Algorithm

input: $a \in \mathbb{Z}$, $d \in \mathbb{Z}^+$

output: q, r s.t.

$a = dq + r$ and

$0 \leq r < d$

$q := a / d;$

$r := a \% d;$

Input: a (non-neg int),

d (pos. int)

Algorithm:

```
q := 0;  r := a;
while (r ≥ d)
    r := r - d
    q := q + 1
end while
```

Output: q, r

GCD : integers a, b $\quad a \neq 0$
$b \neq 0$

$\gcd(a, b) = d$
s.t.
$d \in \mathbb{Z}$

1) d divides both a and b. $\quad d \mid a, d \mid b$

2) $\forall c \in \mathbb{Z}$, if $c \mid a$ and $c \mid b$, then $c \leq d$.

Lemma

If r is a pos. int, then

$$\gcd(r,0) = r$$

1. $r \mid r$ and $r \mid 0$
2. nothing greater than r divides r.

Lemma: If $a, b \in \mathbb{Z}$ and $b \neq 0$ an q, r are non-neg. int s.t.

$$a = bq + r$$

then

$$\gcd(a,b) = \gcd(b,r)$$

Euclidean Algorithm

Input: A, B $\qquad$ gcd(A,B)

Algorithm

$a := A$, $b := B$, $r := B$

while ($b \neq 0$)

$r := a \bmod b$ (see div. alg)

$a := b$

$b := r$

end while

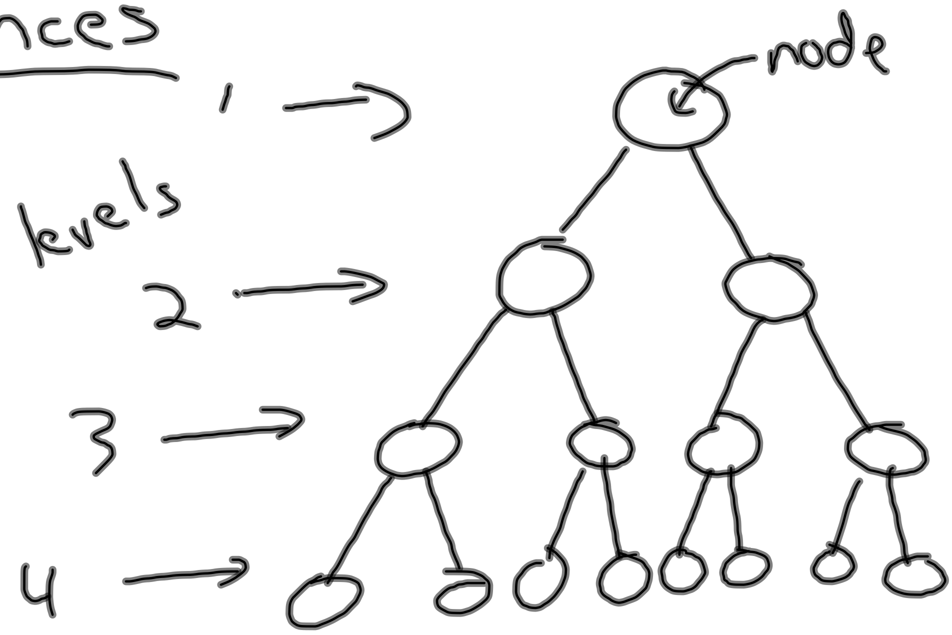
$gcd := a$

Output: gcd

```
int gcd(int A, int B){
    if(B==0)
        return A;
    else
        return gcd(B,
                   A%B);
}
```

$\underbrace{A \% B}_{0..B-1}$

Sequences



Level:	1	2	3	4	5	....
Nodes:	1	2	4	8	16	

$$f(\ell) = \cancel{x?}\ \cancel{\ell}\ 2^{(\ell-1)}$$ closed form

$$N_\ell = \begin{cases} 2 \cdot N_{\ell-1} & \ell > 1 \\ 1 & \ell = 1 \end{cases}$$ recursive

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, -\frac{1}{36}, \ldots$$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$$a_k = \frac{(-1)^{k+1}}{k^2}$$

Alternating sequence

$$-1, 1, -1, 1, -1, 1 \ldots$$

$$c_j = (-1)^j$$