

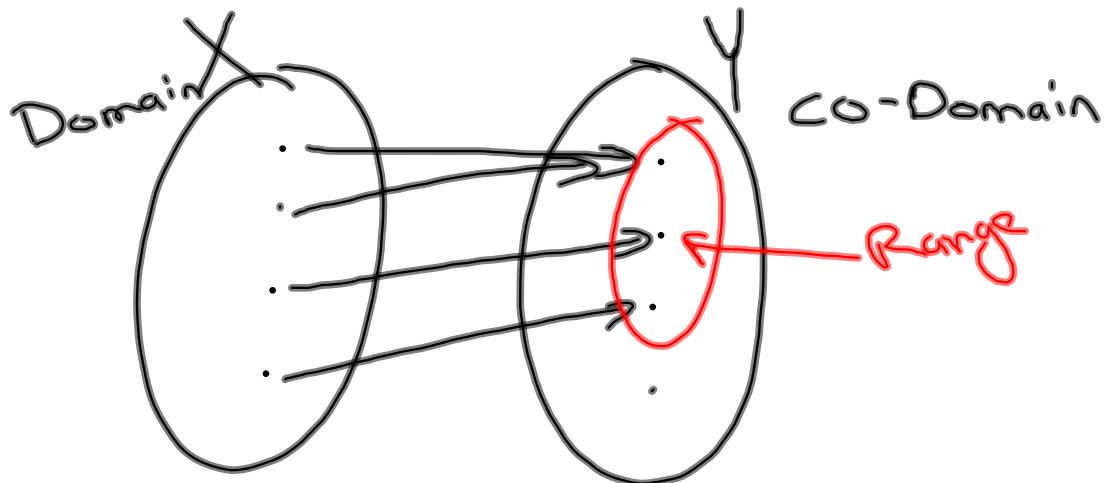
functions

$f: X \rightarrow Y$ mapping

1. $f(x)$ is defined $\forall x$

2. each x maps to exactly 1 y

if $f(x) = y$ and $f(x) = z$
then $y = z$



Equality $f = g$ iff
 $f(x) = g(x) \forall x \in X$

$$f(x) = |x| \quad \forall x \in \mathbb{R}$$

$$g(x) = \sqrt{x^2} \quad \forall x \in \mathbb{R}$$

Does $f = g$?

Disprove: find counter example

Prove: direct proof (algebra)

contradiction:

suppose $f \neq g$

so $\exists x$ s.t. $f(x) \neq g(x)$

Identity function of set X

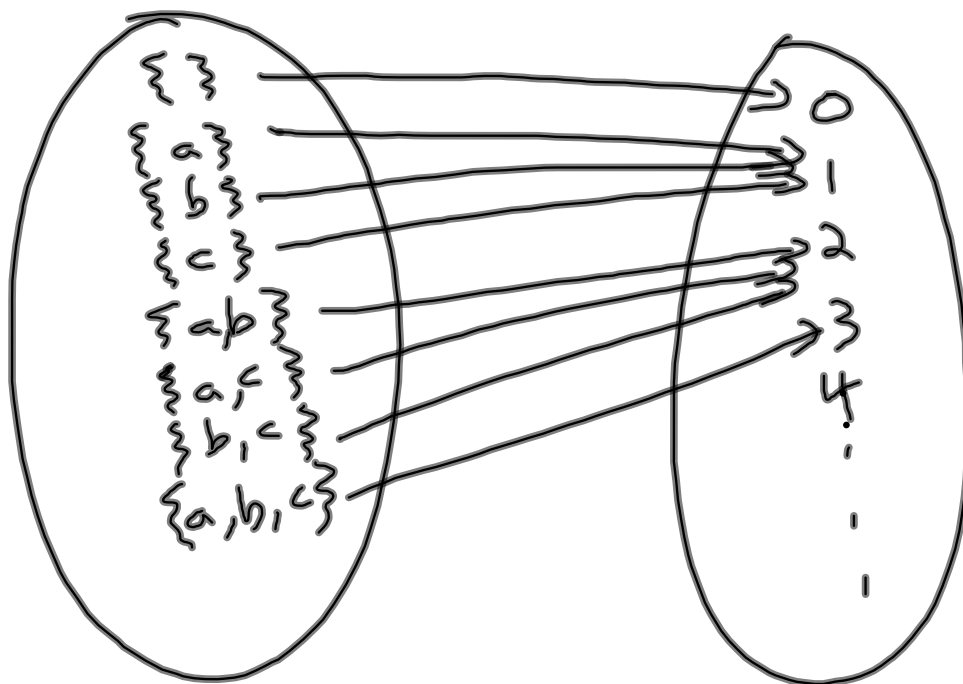
$$i_X: X \rightarrow X$$

$$i_X(x) = x \quad \forall x \in X$$

$$F: \mathcal{P}(\{a, b, c\}) \rightarrow \mathbb{Z}^{\text{non neg}}$$

set of all subsets

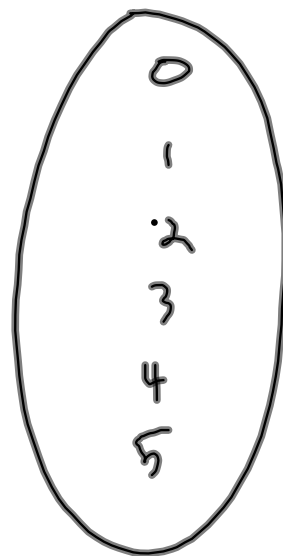
$F(X)$ = number of elements
in set X



$S =$ strings of a's and b's

$$G: S \rightarrow \mathbb{Z}^{\text{nonreg}}$$

$$G(s) = \# \text{ of a's in } s$$



Let $f: X \rightarrow Y$

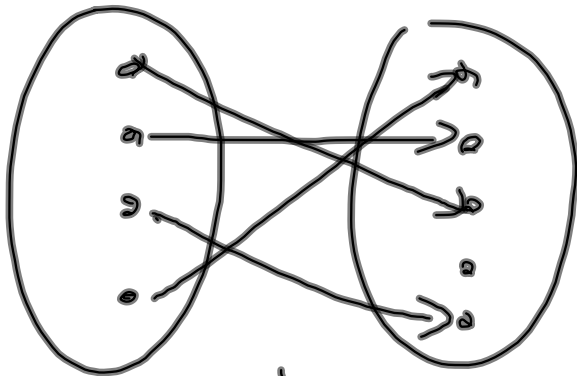
f is one-to-one (injective)

iff for all $x_1, x_2 \in X$

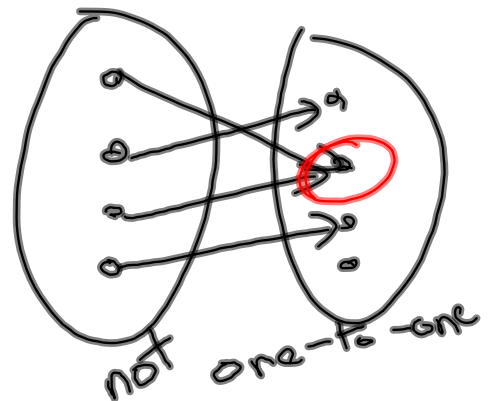
if $f(x_1) = f(x_2)$

then $x_1 = x_2$

equiv. if $x_1 \neq x_2$ then
 $f(x_1) \neq f(x_2)$



one-to-one



not one-to-one

Prove f is one-to-one

suppose $x_1, x_2 \in X$
such that $f(x_1) = f(x_2)$

show $x_1 = x_2$

$$f(x) = 4x - 1$$

suppose $f(x_1) = f(x_2)$

$$\text{so } 4x_1 - 1 = 4x_2 - 1$$

$$\frac{4x_1}{4} = \frac{4x_2}{4}$$
$$\boxed{x_1 = x_2}$$

prove a function is not one-to-one
find elements $x_1, x_2 \in X$
so that $f(x_1) = f(x_2)$
but $x_1 \neq x_2$

e.g.

$$g(x) = x^2$$

$$x \in \mathbb{R}$$

$$x_1 = -2$$

$$x_2 = 2$$

$$g(x_1) = 4$$

$$g(x_2) = 4$$

for $f: X \rightarrow Y$ f is onto
(surjective) iff given any
element $y \in Y$ it is possible
to find an $x \in X$ s.t
 $f(x) = y$

onto: $\forall y \in Y, \exists x \in X$ s.t. $f(x) = y$

not onto: $\exists y \in Y$ s.t. $\forall x \in X, f(x) \neq y$

Prove onto:
suppose $y \in Y$
show there is $x \in X$ s.t. $f(x) = y$

Prove not onto:
find $y \in Y$ s.t. $y \neq f(x) \forall x \in X$

$$h: \mathbb{Z} \rightarrow \mathbb{Z}$$

$$h(n) = 4n - 1 \quad \forall n \in \mathbb{Z}$$

is h onto?

$$\text{Let } y = 5$$

try to find x s.t. $h(x) = y$

$$4x - 1 = 5$$

$$4x = 6$$

$$x = \frac{6}{4}$$

not onto