

Algorithm  
input: X algorithm  
D data (for input to X)

output:  
"halts" if X running  
on D ends

Halting  
Problem

"loops forever" if X running  
on D runs  
forever

There is no such algorithm

Proof: (by contradiction)

Suppose there is, call it

$\text{CheckHalt}(X, D)$   
(defined on previous page)

note it is possible to run

$\text{CheckHalt}(X, X)$

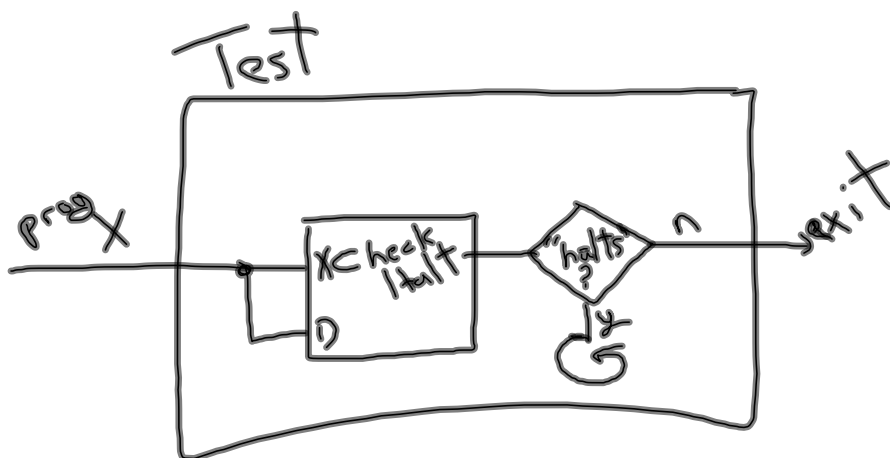
Define a second algorithm

$\text{Test}(X)$ ,  $X$  is a program

$\text{Test}(X)$ :

loops forever iff  $\text{CheckHalt}(X, X)$   
prints "halts"

or  
stops iff  $\text{CheckHalt}(X, X)$   
prints "loops forever"



Now run  $\text{Test}(\text{Test})$

If it terminates in a finite # of steps then  $\text{CheckHalt}(\text{Test}, \text{Test})$  printed "loops forever"

If it loops forever then  $\text{CheckHalt}(\text{Test}, \text{Test})$  printed "halts"

so  $\text{Test}(\text{Test})$  terminates and loops forever which is a contradiction.

so  $\text{CheckHalt}$  does not exist. QED

if and only if proofs  
(iff)

$P \text{ iff } Q \equiv \text{if } P \text{ then } Q \text{ and}$   
 $(2 \text{ proofs}) \text{ if } Q \text{ then } P$