Algorithm
input: X algorithm
D data (for input to)

output:
"halts" if X running
on D ends

"loops forever" if X running
on D runs forever

Halting Problem
There is no such algorithm.

Proof: (by contradiction)

Suppose there is, call it $\text{CheckHalt}(X, D)$.

(note it is possible to run $\text{CheckHalt}(X, X)$)

Define a second algorithm $\text{Test}(X)$, $X$ is a program

$\text{Test}(X)$:

loops forever iff $\text{CheckHalt}(X, X)$

prints "halts"

or

stops iff $\text{CheckHalt}(X, X)$

prints "loops forever"
Now run $\text{Test}(\text{Test})$

If it terminates in a finite # of steps then
$\text{CheckHalt}(\text{Test}, \text{Test})$ printed "loops forever"

If it loops forever, then
$\text{CheckHalt}(\text{Test}, \text{Test})$ printed "halts"

so $\text{Test}(\text{Test})$ terminates and loops forever which is a contradiction.

so $\text{CheckHalt}$ does not exist. $\quad \Box$
if and only if proofs (iff)

\[ p \iff q \equiv \text{if } p \text{ then } q \quad \text{and} \quad \text{if } q \text{ then } p \]

(2 proofs)