$\underline{\text{II}}$ $X = A \cup (B \cap C)$

$Y = (A \cup B) \cap (A \cup C)$

show $\cancel{X \subseteq Y}$ $Y \subseteq X$

case $\underline{1}$: $x \in A$ and $x \in Y$

show $x \in A^{\checkmark}$ or $x \in B \cap C$

by def of $\cup$

case 2: $x \notin A$ and $x \in Y$

show $\cancel{x \in A}$ or $x \in B \cap C$

since $x \in Y$, $x \in A \cup B$ and $x \in A \cup C$.

since $x \in A \cup B$, $x \in A$ or $x \in B$

but $x \notin A$ so $x \in B$

same w/ $x \in A \cup C$, so $x \in C$

so $x \in B$ and $x \in C$

$\therefore x \in B \cap C$

so $x \in A$ or $x \in B \cap C$

$\therefore x \in A \cup (B \cap C)$

$X \subseteq Y$ and $Y \subseteq X$ so

$X = Y$

QED

Prove a set is empty $X = \emptyset$

1. suppose $x \in X$
2. show a contradiction

$\forall$ sets A, B, C

if $A \subseteq B$ and $B \subseteq C^c$

then $A \cap C = \emptyset$

suppose $x \in A \cap C$

so $x \in A$ and $x \in C$

since $x \in A$ we know $x \in B$

so $x \in C^c$.

so $x \notin C$ which is a contradiction.

Algebraic proofs

Show $(A \cup B) - C = (A - C) \cup (B - C)$

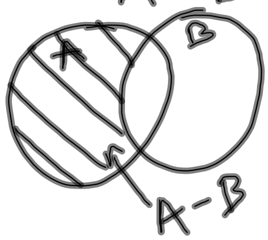
$(A \cup B) - C = (A \cup B) \cap C^c$ set difference Law $A - B = A \cap B^c$

$= C^c \cap (A \cup B)$

$= (C^c \cap A) \cup (C^c \cap B)$

$= (A \cap C^c) \cup (B \cap C^c)$

$= (A - C) \cup (B - C)$



Boolean Algebra

a set B w/ operations $+$ and $\cdot$

w/ the following properties

1. commutativity: $a + b = b + a \quad a \cdot b = b \cdot$
2. associativity: $(a+b)+c = a+(b+c)$
3. distributivity: $a \cdot (b+c) = (a \cdot b) + (a \cdot$

   $a + (b \cdot c) = (a+b) \cdot (a+$
4. identity: $\exists$ distinct elements $0, 1 \in B$ s.t.

   $a + 0 = a \quad b \cdot 1 =$
5. complement: for each $a \in B$ $\exists$ $\bar{a} \in B$ s.t.

   $a + \bar{a} = 1$ and $a \cdot \bar{a} =$

Russell's Paradox

$$U = \{\text{all sets}\}$$

$$U \in U ?$$

$$S = \{A \mid A \text{ is a set and } A \notin A\}$$

$$S \in S ?$$