5.5 #11

$I(n): \; xy + product = A \cdot B$

suppose $I(k)$, show $I(k+1)$

↳ $x_k y_k + product_k = A \cdot B$

show $x_{k+1} y_{k+1} + product_{k+1} = A \cdot B$

case 1: y is even

$x_{k+1} = x_k \cdot 2$

m_n : minimum number of moves required to more n disks to the destination

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$$m_k = m_{k-1} + 1 + m_{k-1}$$
$$= 2m_{k-1} + 1$$

$m_1 = 1$

$m_2 = 3$

$m_3 = 7$

$m_4 = 15$

$m_5 = 31$

Count bit strings that do not contain 11

S_n : The number of bit strs of length n that don't contain 11

S_1 : 0, 1 — 2

S_2 : 00, 01, 10, ~~11~~ — 3

S_3 : 000, 001, 010, ~~011~~, 100, 101, ~~110~~, ~~111~~ — 5

S_4 : 0000, 0001, 0010, ~~0011~~,
0100, 0101, ~~0110~~, ~~0111~~,
1000, 1001, 1010, ~~1011~~,
~~1100~~, ~~1101~~, ~~1110~~, ~~1111~~ — 8

S_5 : 12?

S_4 {
00000, 00001, 00010, 0~~0011~~,
00100, 00101, ~~0110~~, ~~0111~~,
01000, 01001, 01010, ~~1011~~,
~~1100~~, ~~1101~~, ~~1110~~, ~~1111~~

S_3 {
10000, 10001, 10010, ~~0011~~,
10100, 10101, ~~0110~~, ~~0111~~,
~~11000~~, ~~11001~~, ~~11010~~, ~~1011~~,
~~1100~~, ~~1101~~, ~~1110~~, ~~1111~~

$$S_n = S_{n-1} + S_{n-2}$$

fibonacci numbers

1. rabbits don't procreate their first month, after that. 1 new pair/month
2. rabbits don't die

$F_0 = 1$ prior to first month

$F_1 = 1$

$F_2 = 2$ $F_n = F_{n-1} + F_{n-2}$

$F_3 = 3$

$F_4 = 5$

$F_5 = 8$

$a_k = a_{k-1} + 2$

$a_0 = 1$

$a_1 = 1 + 2 = 3$

$a_2 = 3 + 2 = 5$ $\quad (1+2)+2$

$a_3 = 5 + 2 = 7$ $\quad ((1+2)+2)+2$

$a_4 = 7 + 2 = 9$

$a_5 = 9 + 2 = 11$

$a_n = 2n + 1$ conjecture

$n \geq 0$

Induction

Basis: $n = 0$

$a_0 = 2(0) + 1 = 1$

Induction:

suppose $a_k = 2(k) + 1$

show $a_{k+1} = 2(k+1) + 1$

$a_{k+1} = a_k + 2$ by def of a

$= (2(k)+1) + 2$ subst. ind. hyp

$= 2k + 1 + 2$

$= 2k + 2 + 1$

$= 2(k+1) + 1$

Arithmetic Sequence

$\exists$ a constant d s.t

$a_k = a_{k-1} + d$ for all $k \geq 1$

$a_n = a_0 + d \cdot n \quad \forall n \geq 0$

Geometric Sequence

$$a_k = r \cdot a_{k-1} \qquad r \neq 0$$

$$a_0 = a$$

$$a_1 = r \cdot a$$

$$a_2 = r^2 \cdot a$$

$$a_3 = r^3 \cdot a$$

$$a_n = r^n \cdot a$$

Basis $n=0$

$$a_0 = r^0 \cdot a = 1 \cdot a$$

Ind.

suppose $a_k = r^k \cdot a$

show $a_{k+1} = r^{k+1} \cdot a$

$$a_{k+1} = r \cdot a_k = r \cdot r^k \cdot a$$
$$= r^{(k+1)} \cdot a$$