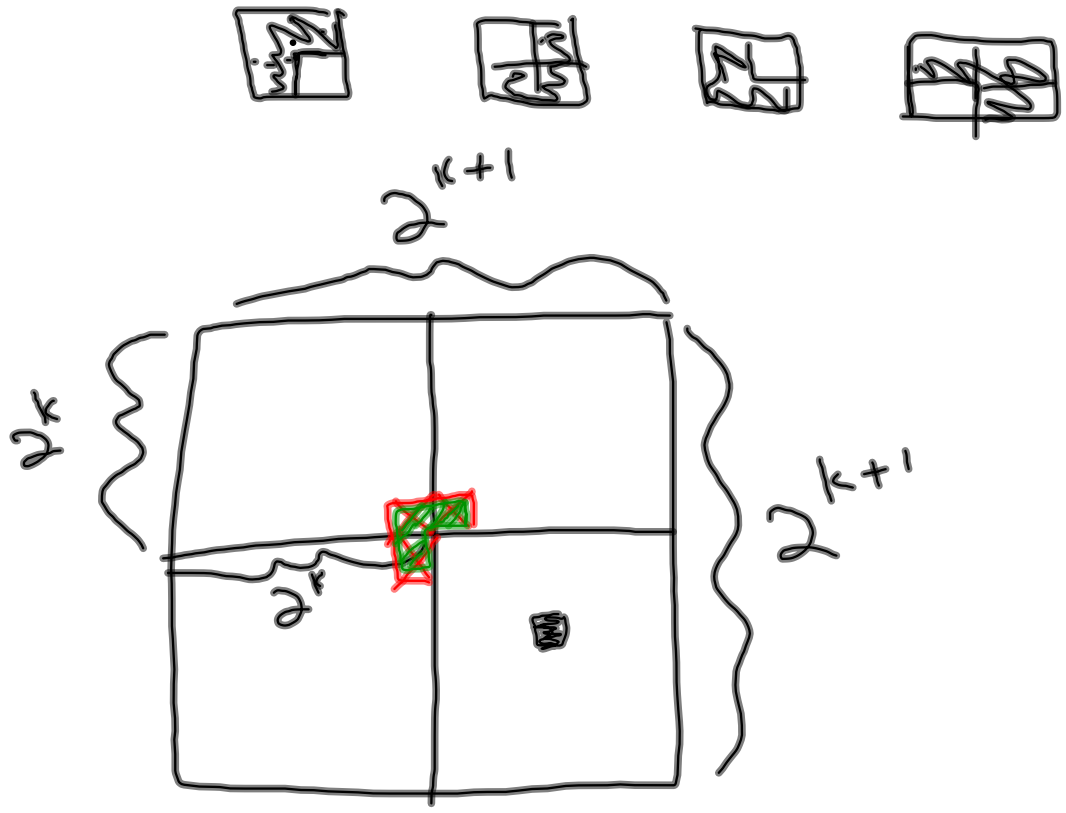
2^{k+1}
2^k
2^{k+1}
2^k

Strong induction

show $P(n)$ for $n \geq a$

Basis: show $P(a)$

Induction: suppose $P(i)$
$a \leq i < k$
show $P(k)$

Prove that every integer greater than 1 is divisible by a prime number.

Basis: $n = 2$

Suppose all integers i, $2 \leq i < k$
i is divisible by a prime num.

Show k is divisible by a prime number.

case I: k is prime, so
$P(k)$ is true.

case II: k is not prime
so $k = m \cdot n$, $m, n \in \mathbb{Z}^+$
and $m \neq 1, n \neq 1$
$2 \leq m < k$
$2 \leq n < k$

Since $2 \leq m, n < k$ and are integers they must be divisible by prime numbers, by the induction hypothesis.

$\therefore$ k is divisible by some prime number

sequence

$a_1 = 0$

$a_2 = 2$

$a_k = 3 \cdot a_{\lfloor k/2 \rfloor} + 2$ for $k \geq 3$

Prove a_n is even for $n \geq 1$

Basis: $a_1 = 0$ is even
$a_2 = 2$ is even

Induction: Suppose for $3 \leq i < k$

a_i is even

Show a_k is even

$$a_k = 3 a_{\lfloor k/2 \rfloor} + 2$$

now $a_{\lfloor k/2 \rfloor}$ is even
by the ind. hyp.

$r = 3 \cdot a$ is even

$r + 2$ is even

so a_k is even

Algorithm Correctness

If it is given correct input
then it produces correct output.

$P \rightarrow Q$

pre-conditions – assert that input is correct

post-conditions – assert that output is correct

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precond. P

Algorithm

postcond: Q

if P then Q

P

A_1 part of our algorithm

P_1 - post conditions on A_1
pre-cond. for A_2

A_2

P_2

$\vdots$

A_n

$P_n = Q$

to prove $P \to Q$

$P \to P_1$

$P_1 \to P_2$

$P_2 \to \dots$

$P_{n-1} \to P_n$ which is Q

Correctness of loops
(method of loop invariants)