$$\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$$

for all integers $n \geq 2$

<u>Basis</u>: $n=2$

LHS: $1-\frac{1}{2^2}=\frac{3}{4}$

RHS: $\frac{2+1}{2\cdot 2}=\frac{3}{4}$ LHS = RHS ✓

<u>Induction</u>: Suppose it is true for $n=k$, $k \geq 2$

[Show it must be true for $n=k+1$]

Suppose $\prod_{i=2}^{k}\left(1-\frac{1}{i^2}\right)=\frac{k+1}{2k}$

Show $\prod_{i=2}^{k+1}\left(1-\frac{1}{i^2}\right)=\frac{(k+1)+1}{2(k+1)}$

$$\prod_{i=2}^{k+1}\left(1-\frac{1}{i^2}\right)=\left[\prod_{i=2}^{k}\left(1-\frac{1}{i^2}\right)\right]\cdot\left(1-\frac{1}{(k+1)^2}\right)$$

from the induction hypothesis substitute for $\prod_{i=2}^{k}\left(1-\frac{1}{i^2}\right)$

so $$=\left[\frac{k+1}{2k}\right]\cdot\left(1-\frac{1}{(k+1)^2}\right)$$

$$=\frac{k+1}{2k}-\frac{\cancel{k+1}}{2k}\cdot\frac{1}{(k+1)^{\cancel{2}}}$$

$$=\frac{k+1}{2k}-\frac{1}{2k(k+1)}$$

$$=\frac{(k+1)(k+1)-1}{2k(k+1)}$$

$$=\frac{(k^2+2k+1)-1}{2k(k+1)}$$

$$=\frac{k^2+2k}{2k(k+1)}=\frac{\cancel{k}(k+2)}{2\cancel{k}(k+1)}$$

$$=\frac{k+2}{2(k+1)}=\frac{(k+1)+1}{2(k+1)}$$

for all integers $n \geq 1$,

$2^{2n} - 1$ is divisible by 3

Basis: $n = 1$

$2^{2(1)} - 1 = 4 - 1 = 3$

Hyp: Suppose $2^{2k} - 1$ is divisible by 3.

Show $2^{2(k+1)} - 1$ is divisible by 3

$$
\begin{aligned}
2^{2(k+1)} - 1 &= 2^{2k+2} - 1 \\
&= 2^{2k} \cdot 2^2 - 1 \\
&= 2^{2k} \cdot 4 - 1 \\
&= 3 \cdot 2^{2k} + \underbrace{2^{2k} - 1}_{\text{ind. hyp}}
\end{aligned}
$$

This is divisible by 3.

$= 3 \cdot 2^{2k} + 3r$ where $r \in \mathbb{Z}$ (def of divid

$= 3(2^{2k} + r)$

Let $s = 2^{2k} + r$ so $s \in \mathbb{Z}$

$= 3s$ where $s \in \mathbb{Z}$

$n \geq 3, \quad 2n+1 < 2^n$

Basis $P(3)$ $\quad 2(3)+1 = 7$

$2^3 = 8 \qquad 7 < 8$

Ind.

Suppose $\quad 2k+1 < 2^k$

Show $\quad \underline{2(k+1)+1} < 2^{k+1}$

$$2(k+1)+1 = 2k+2+1$$

$$= (2k+1)+2$$

ind. hyp.

$$< 2^k + 2 < 2^k + 2^k$$

since $2 < 2^k$

$k \geq 3$

$$< 2^k + 2^k$$

$$< 2^k \cdot 2$$

$$< 2^{k+1}$$

so $2(k+1)+1 < 2^{k+1}$

$a_1 = 2$

$a_m = 5 \cdot a_{m-1}$ for $m \geq 2$

$a_2 = 5(a_1) = 5 \cdot 2 = 10$

$a_3 = 5(a_2) = 5(10) = 50$

Show the explicit formula is

$a_n = 2 \cdot 5^{n-1}$ for $n \geq 1$

Basis: $a_1 = 2 \cdot 5^{(1-1)} = 2 \cdot 5^0 = 2$

true by recursive def. of a.

Ind.: suppose $a_k = 2 \cdot 5^{k-1}$

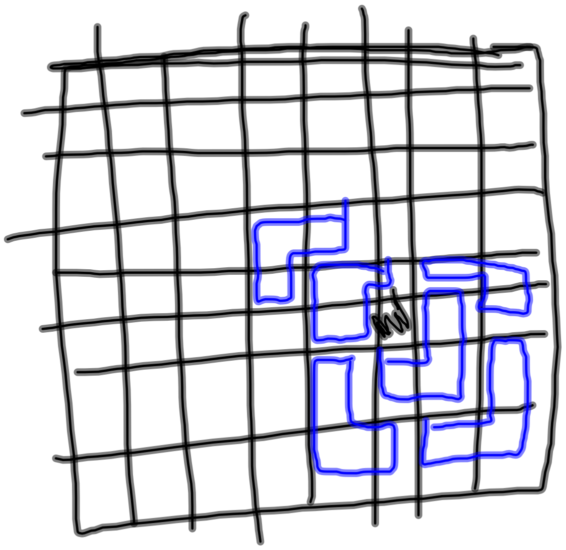
show $a_{k+1} = 2 \cdot 5^{(k-1)+1}$

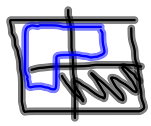
$a_{k+1} = 5 \cdot a_k$ by rec. def of a

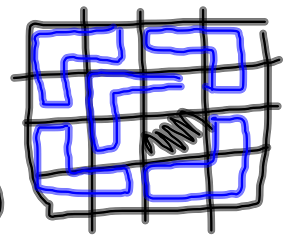
$= 5(2 \cdot 5^{k-1})$ by ind. hyp

$= 2 \cdot 5^k$

$2^k \times 2^k \quad k \geq 1$







one square removed

such a board can be covered w/ L-trominos