

$P \rightarrow q$

$\neg(P \rightarrow q)$

$\neg(P \vee \neg q)$

pick a number

$$\begin{array}{r} 7 \quad x \\ \hline 12 \quad x+5 \\ \hline 48 \quad 4 \cdot (x+5) \\ \hline 42 \quad 4x+20 \\ \hline 21 \quad 2x+7 \\ \hline 7 \quad 2x+7 - 2x \\ = 7 \end{array}$$

add 5

multiply by 4

subtract 6

divide by 2

subtract twice the  
original

## proving universal statements (Direct Proof)

1. Express statement as:

$$\forall x \in D, \text{ if } P(x) \text{ then } Q(x)$$

2. Start by supposing  $x$  is a particular, but arbitrarily chosen element of  $D$  for which  $P(x)$  is true.

3. Show  $Q(x)$  is true

use: definitions, previous results,  
operations from the domain,  
rules of inference.

Show that the sum of two odd integers is even.

		start
		finish
		$\forall m, n \in \mathbb{Z}$ , if $m$ and $n$ are odd, then $m+n$ is even.
$p$	$q$	$p \rightarrow q$
T	T	T
T	F	? $\Leftarrow$
F	T	T
F	F	T

Starting point:  
Suppose  $m, n$  are particular  
but arbitrarily chosen integers  
that are odd.

Show  $m+n$  is even.

Proof:

$m$  is odd.

so  $m = 2r+1$  where  $r$  is an integer. Def. of odd.

$n$  is odd  
so  $n = 2s+1$  where  $s$  is an integer. Def. of odd.

$$\begin{aligned} \text{so } m+n &= (2r+1) + (2s+1) \text{ subst.} \\ &= 2r+2s+2 \\ &= 2(r+s+1) \end{aligned}$$

Let  $k = r+s+1$   
so  $k$  is an integer since  $r, s, 1$  are integers and integers are closed under  $+$

$$m+n = 2k \text{ where } k \text{ is an integer}$$

def. of even

$\therefore m+n$  is even, by def of even.

QED

//  $\square$

Show that The sum of an even  
and an odd number is odd.

$\forall n, m \in \mathbb{Z}$ , if  $n$  is even and  
 $m$  is odd then  $n+m$  is  
odd.

$$n \text{ is even so } n = 2 \cdot r$$

$$m \text{ is odd so } m = 2 \cdot s + 1$$

$$\begin{aligned} n+m &= (2r) + (2s+1) \\ &= 2r + 2s + 1 \\ &= 2(r+s) + 1 \end{aligned}$$

Let  $k = r+s$ .  $k$  is an integer  
so  $n+m = 2k+1$

By def of odd,  $n+m$  is odd

Q.E.D.