Study of Pseudo-magnetic fields in NSR Exotic 5th force experiment

Axial-axial potential

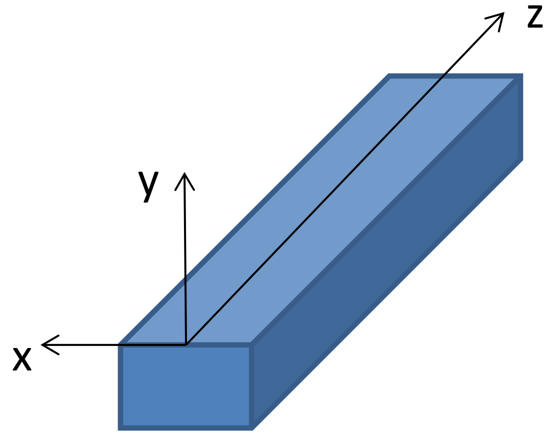
$$V_{AA}^{point}(r) = \frac{g_A^2\,(\hbar c)^2}{16\,\pi\, mc^2}\vec{\sigma}\ \cdot\left(\frac{\vec{v}}{c}\times\frac{\vec{r}}{r}\right)\left(\frac{1}{\lambda_c}+\frac{1}{r}\right)\frac{e^{-r/\lambda_c}}{r}$$

gives rise to pseudo-magnetic field via

$$V = -\frac{1}{2}\gamma_n\hbar\vec{\sigma}\ \cdot\vec{B}$$

Solving for pseudo-magnetic field by equating above equations we get a differential field due to a bulk material

$$dB_{AA} = \frac{-g_A^2\, \hbar c\, N}{\gamma_n 8\, \pi\, mc^2} \begin{bmatrix} v_y(z - z_s) - v_z(y - y_s) \\ v_z(x - x_s) - v_x(z - z_s) \\ v_x(y - y_s) - v_y(x - x_s) \end{bmatrix} \left(\frac{1}{\lambda_c} + \frac{1}{r}\right) \frac{e^{-r/\lambda}}{r^2} dxdydz$$

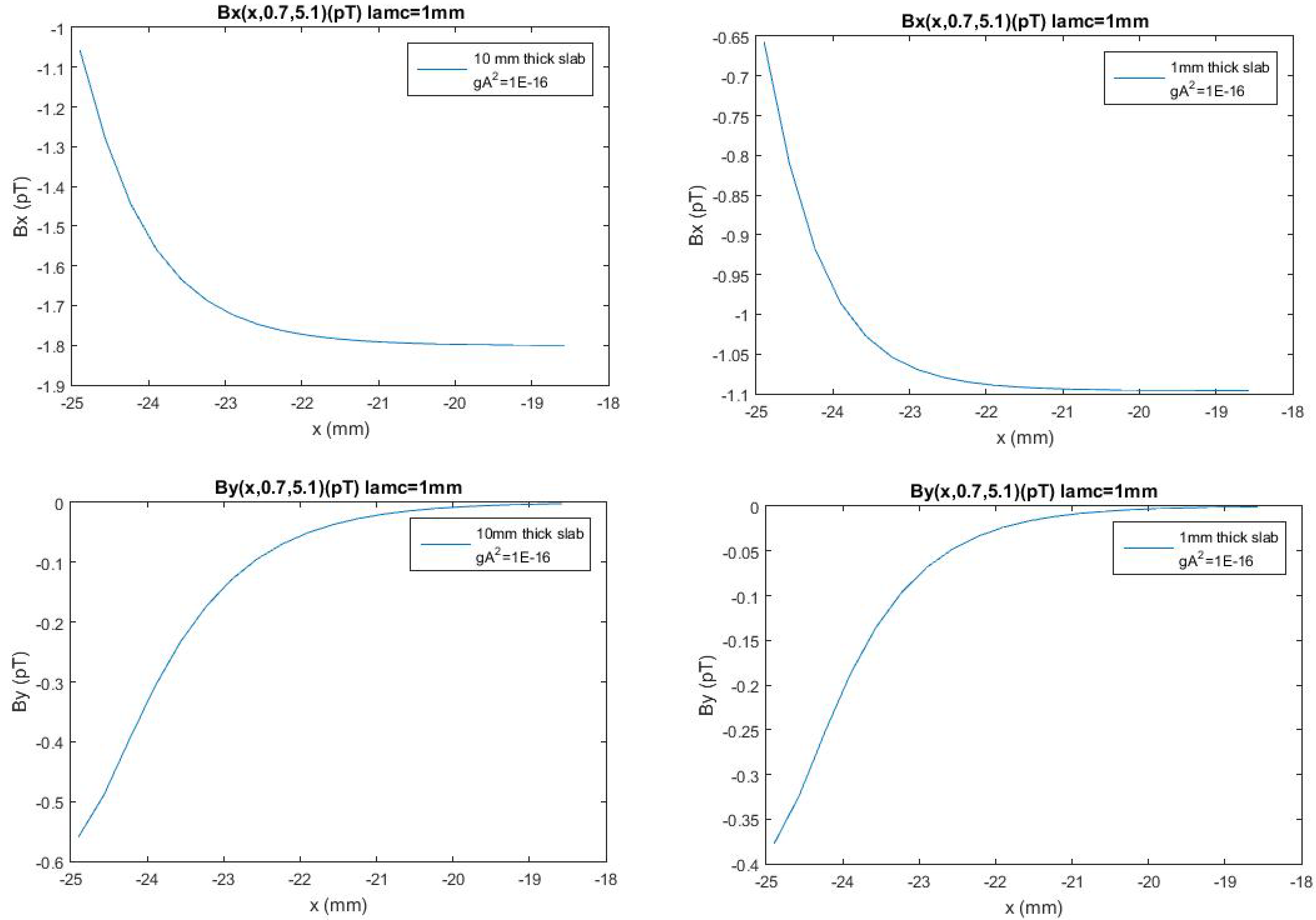


Matlab code sums over slab source points $[x_s y_s z_s]$ to calculate pseudo magnetic field due to finite slab.

Note: For semi-infinite slab with beam parallel to surface, let $\vec{v} = v_z$. This eliminates B_z and symmetry of semi-infinite slab eliminates B_y, leaving only a horizontal pseudo-magnetic field.

Using cylindrical coordinates to integrate over the semi-infinite slab, where $r[0,\infty]$, $\theta[0,2\pi]$, $y[-\infty,0]$, yields the equation from Piegsa and Pignol. [JoP: Conf. Series **340** (2012) 012043, eq. 7], where Δy is the vertical distance from the slab surface to the field point.

$$B_{AA} = \frac{1}{\gamma_n} \frac{g_A^2}{4} N \frac{\hbar c}{mc^2} \lambda_c (\vec{v} \times \widehat{e_y})\, e^{-\Delta y/\lambda_c}$$

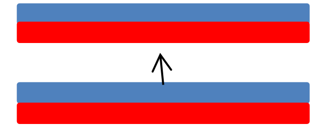


- ✓ Semi-infinite slab approximation with N_{Cu}=1.9*10^{29} /m^3, λ_c=1mm at y=0.7mm, v=660m/s gives B=1.8pT which agrees with a 10-mm thick slab away from edges. 1-mm thick slab gives slightly smaller field. Note that B_z = 0 when v=v_z. Code focuses on corner region of slab, x~-25mm, z~0mm.

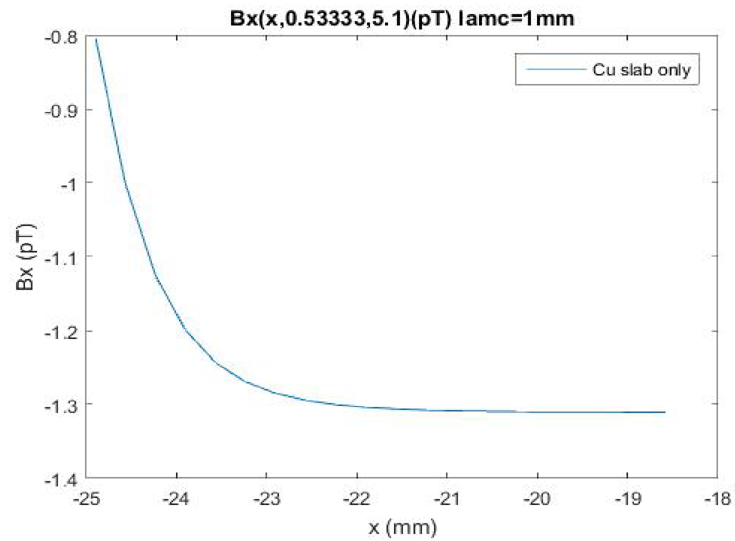
λ_c=1mm

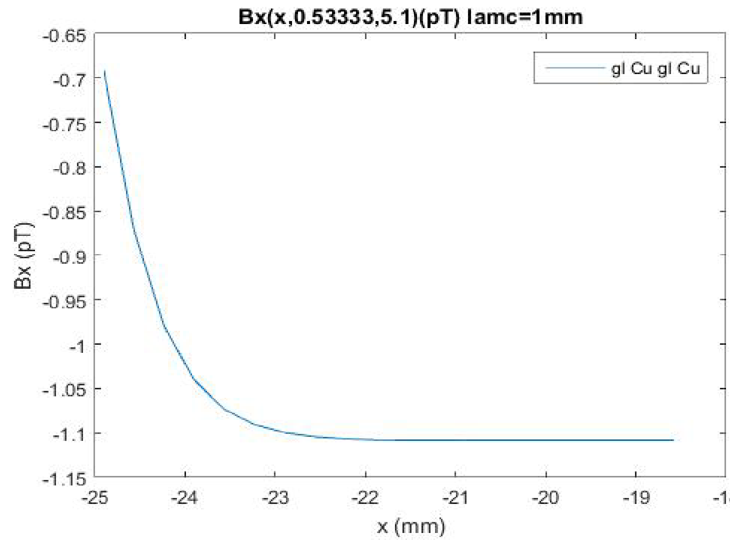
1 slab versus 4

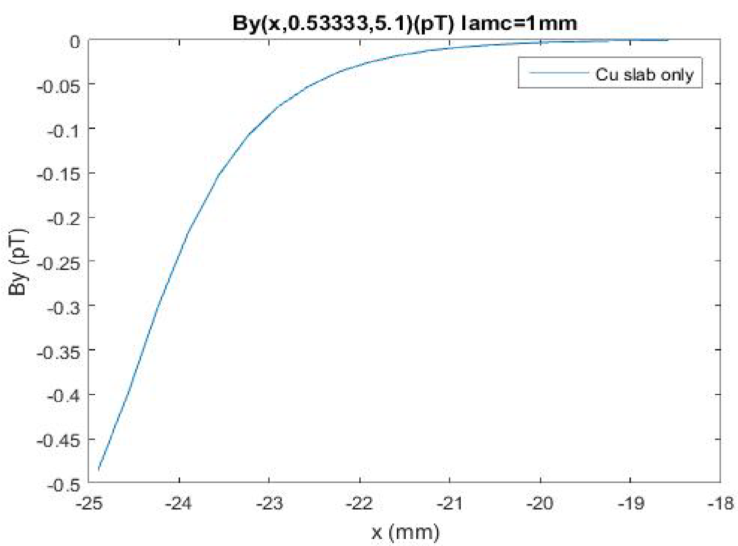
Cu (N=1.9E29/m^3)

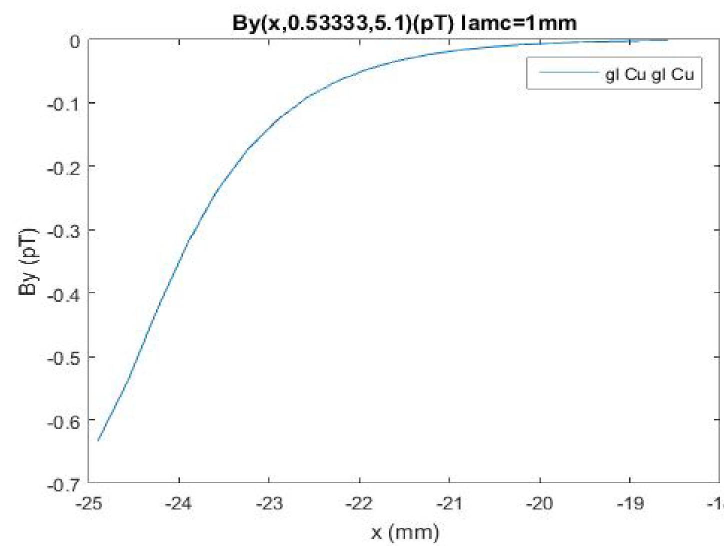


glass (N=0.75E29/m^3)

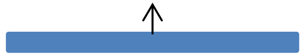






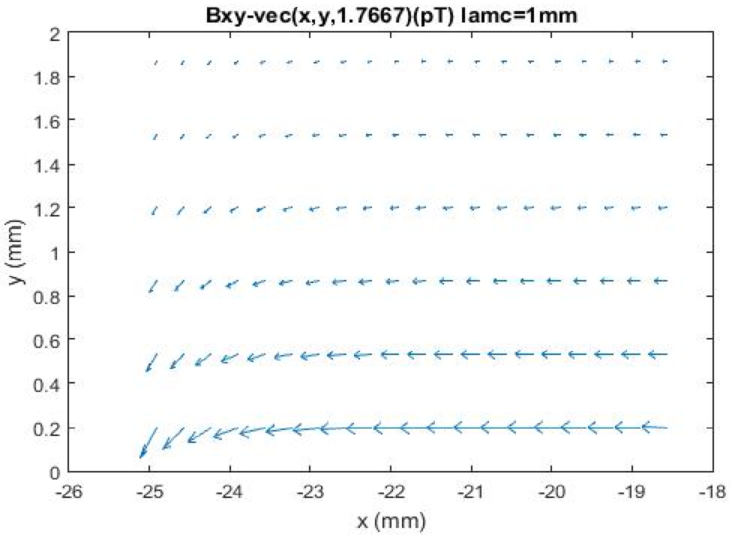


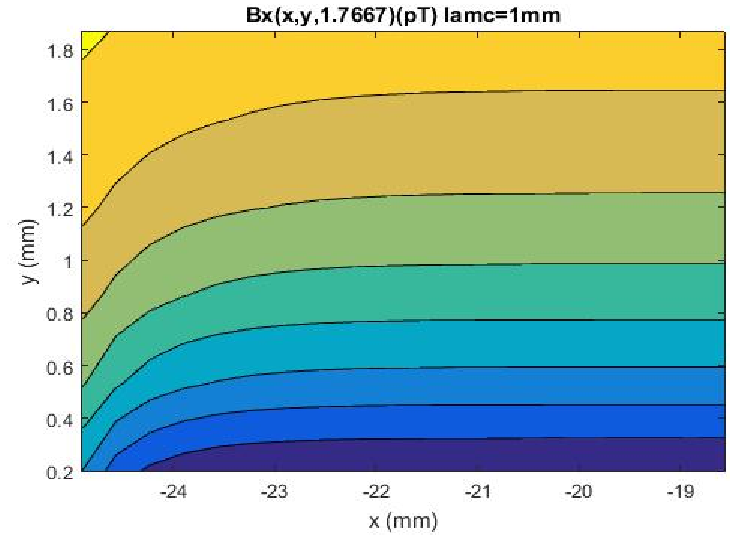
λ_c=1mm

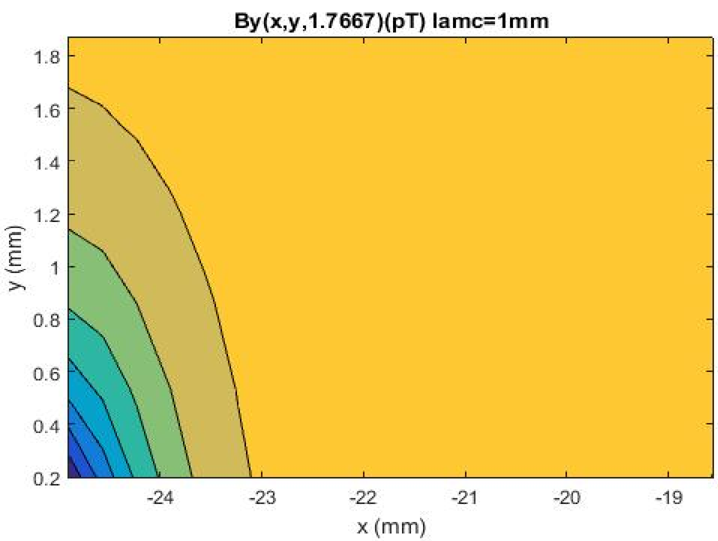


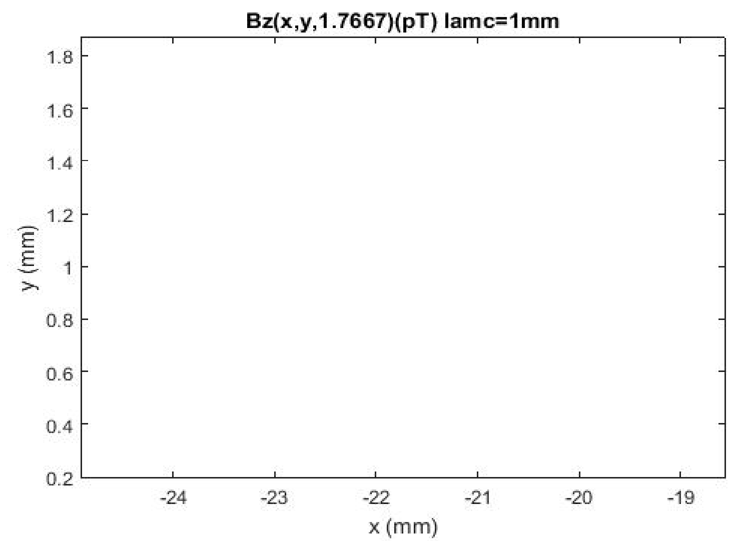
Cu (N=1.9E29/m^3)

1 slab



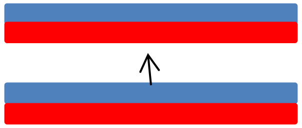






λ_c=1mm

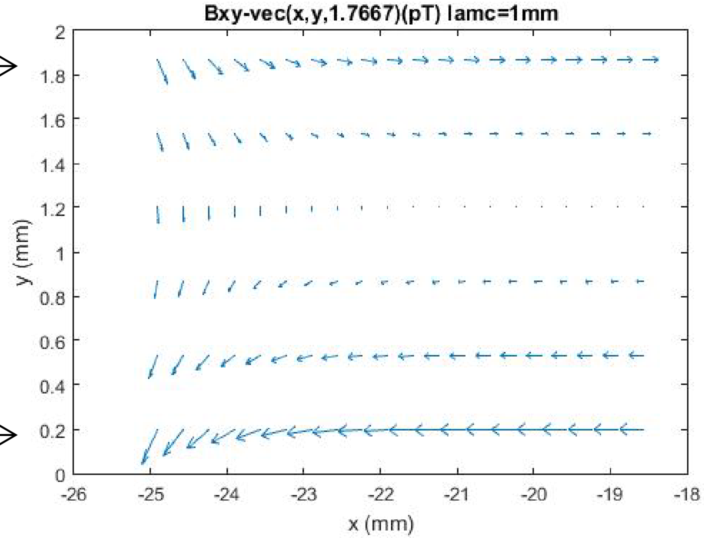
4 slabs

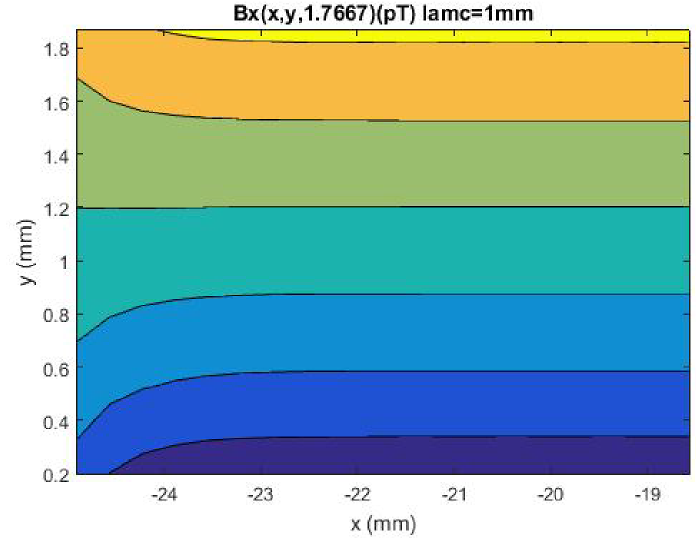


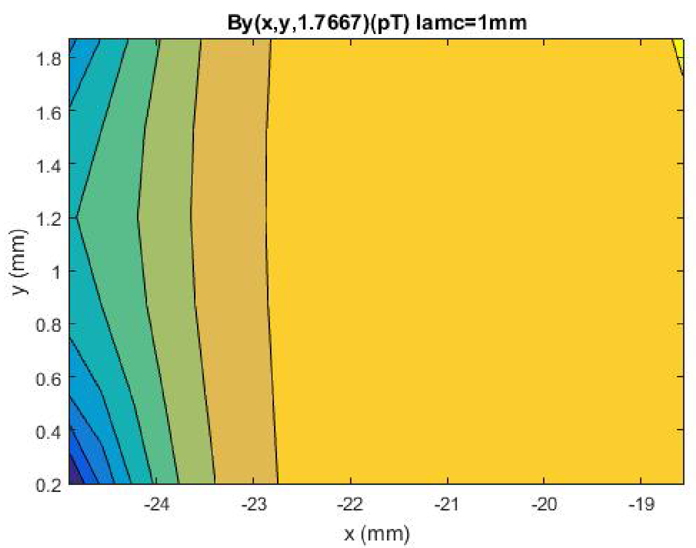
glass (N=0.75E29/m^3)

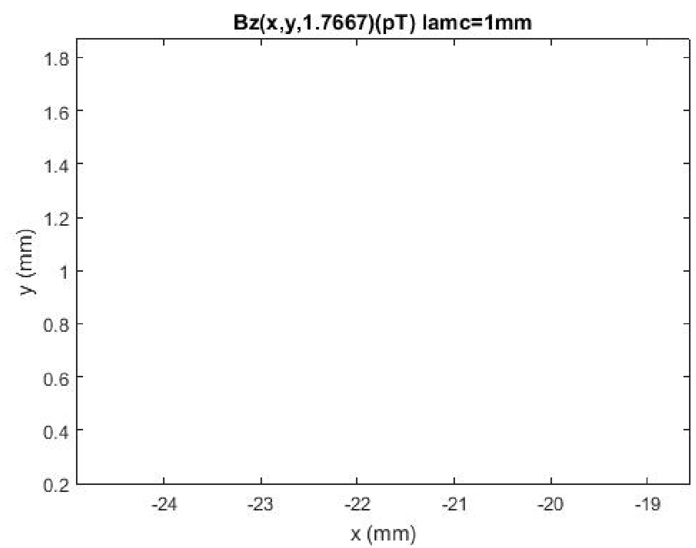
+1.1pT →

-1.8pT →





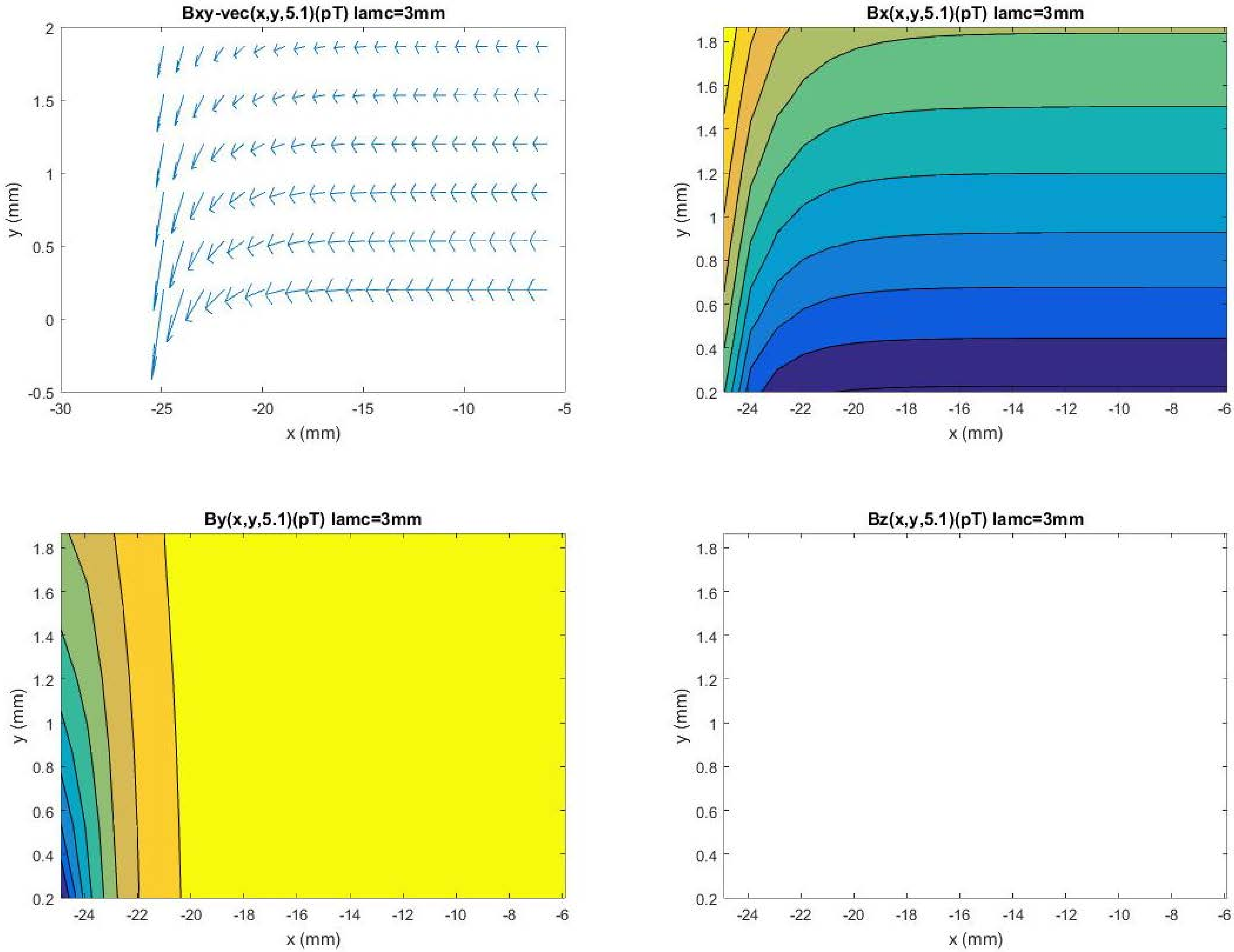




λ_c=3mm

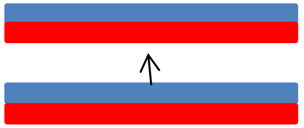
1 slab

Cu (N=1.9E29/m^3)

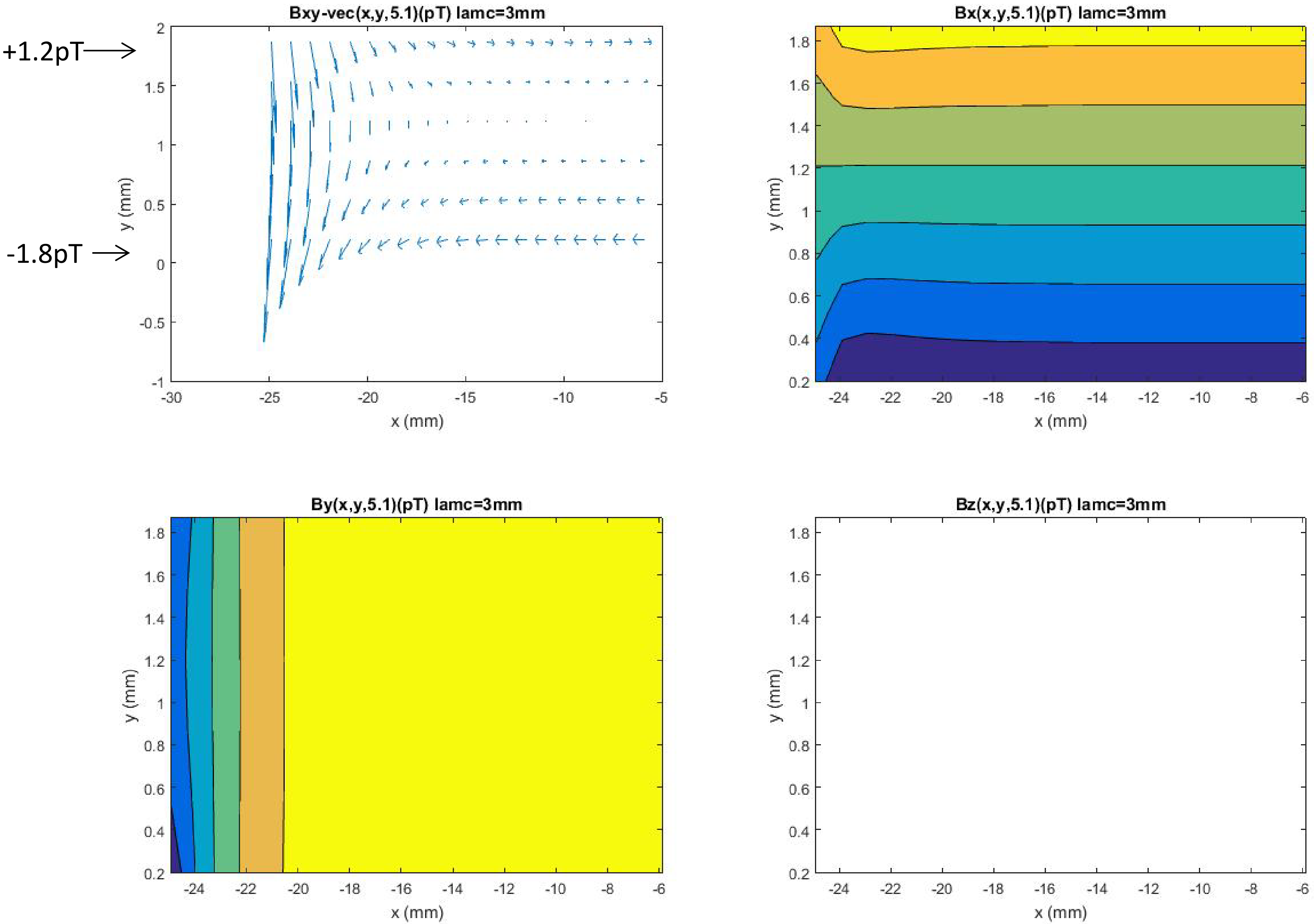


λ_c=3mm

4 slabs

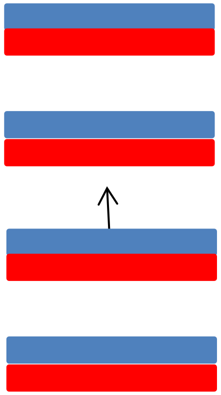


glass (N=0.75E29/m^3)



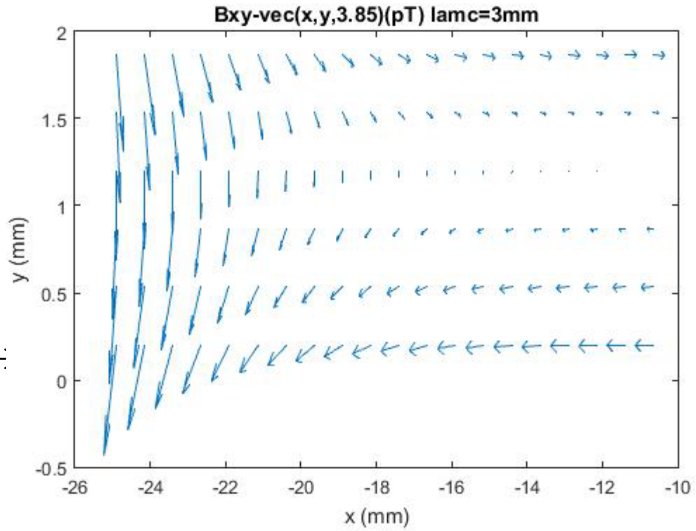
λ_c=3.0mm

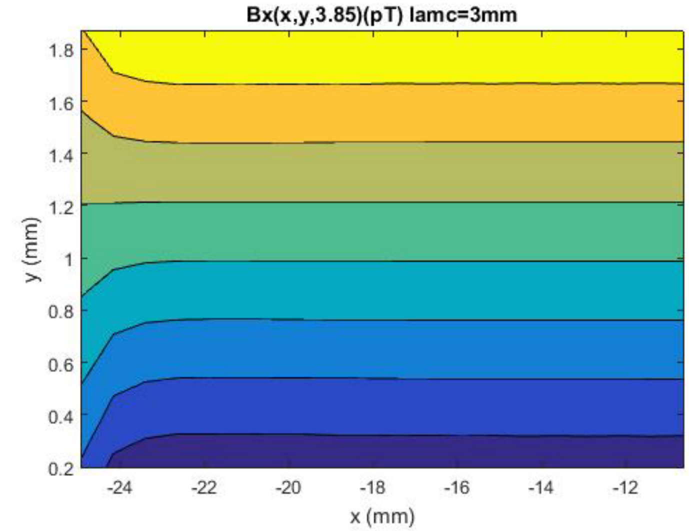
8 slabs

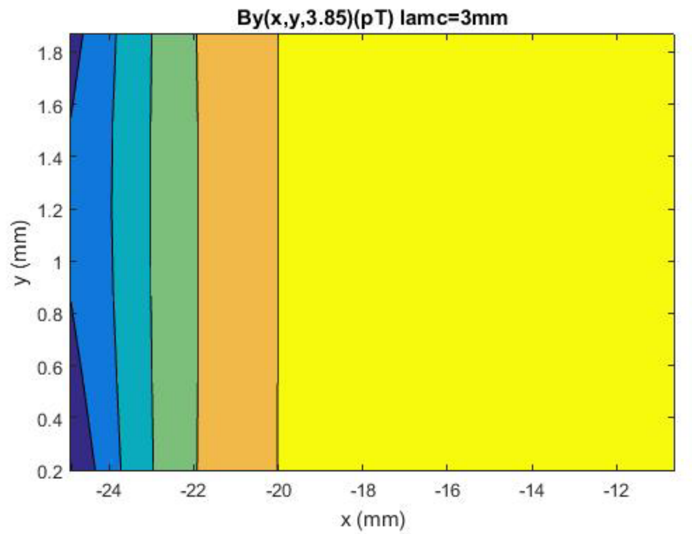


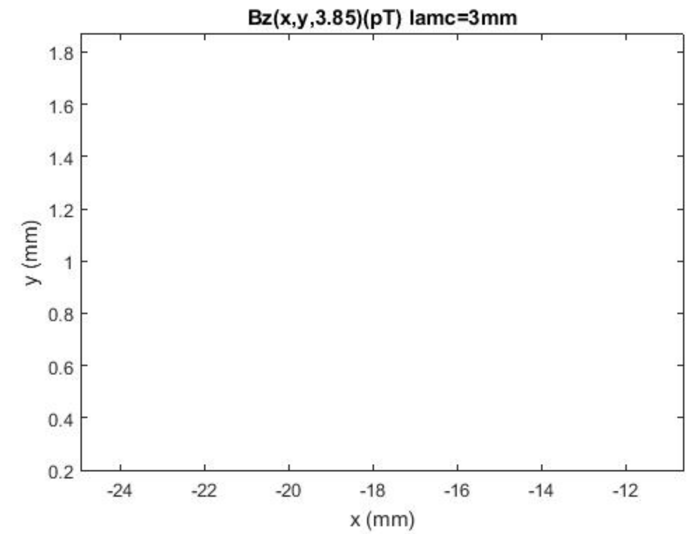
+1.43pT →

-2.25pT



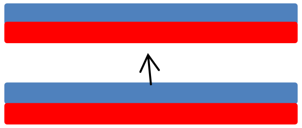






λ_c=0.3mm

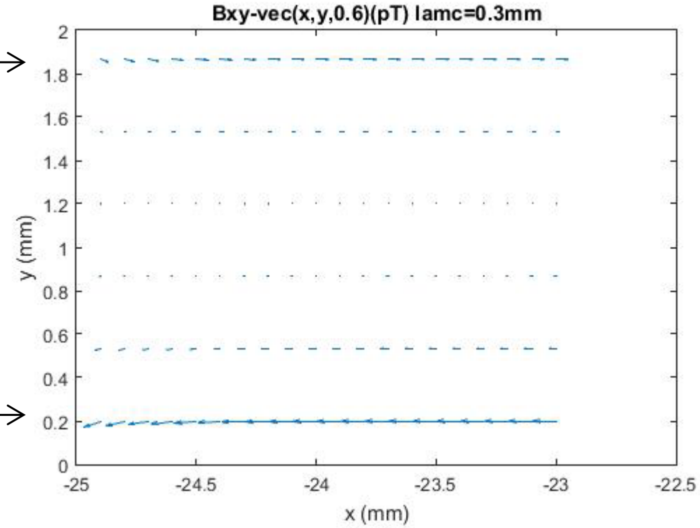
4 slabs

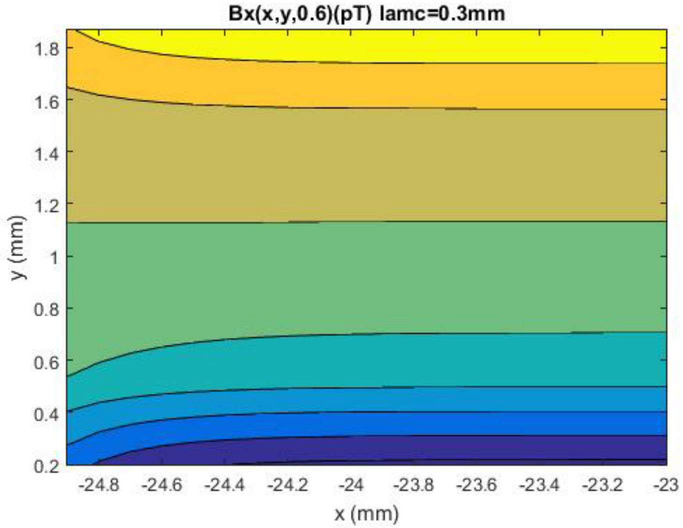


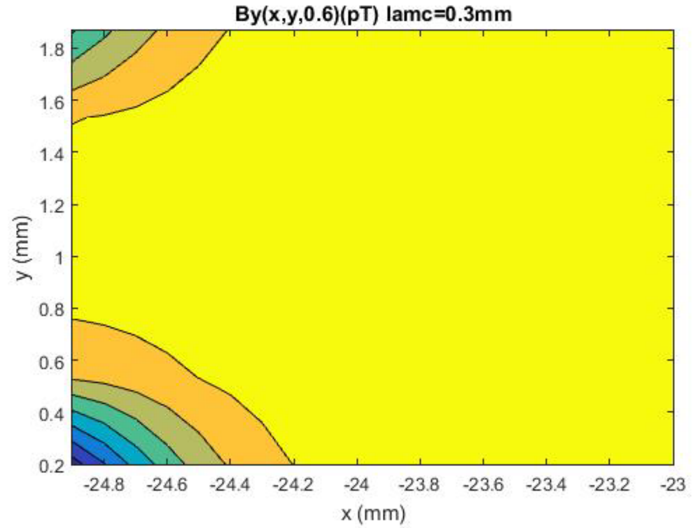
glass ($N=0.75E29/m^3$)

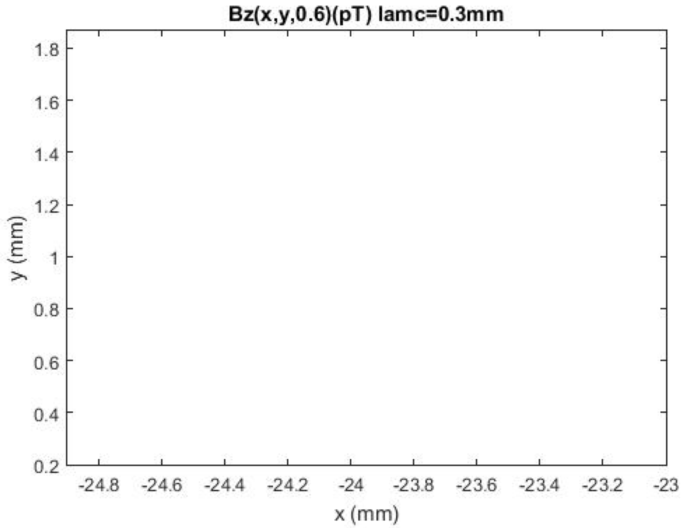
+0.27pT →

-0.51pT →

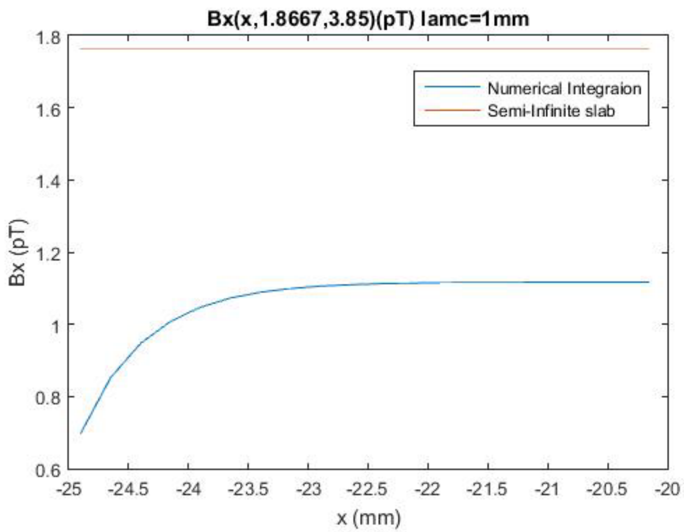


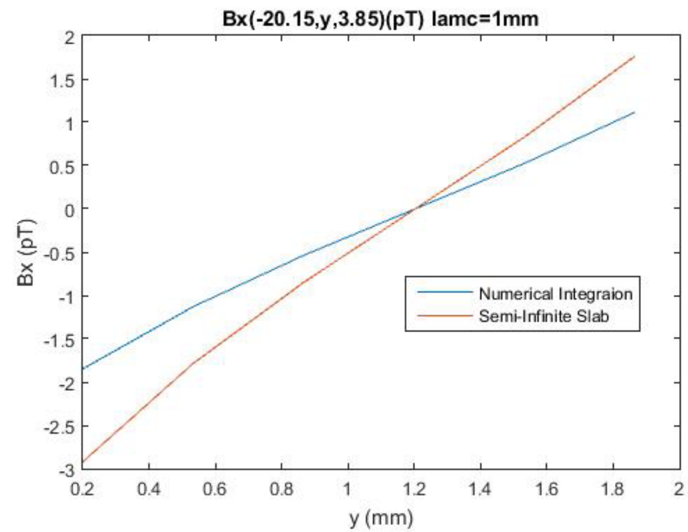


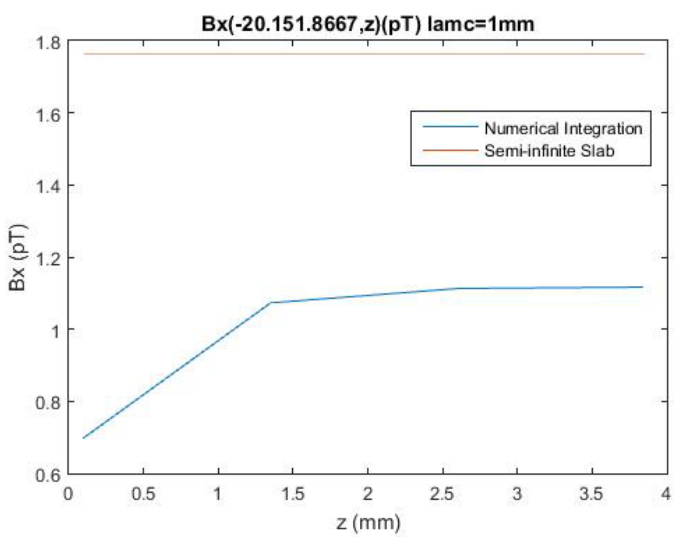




B_x versus x, y, z



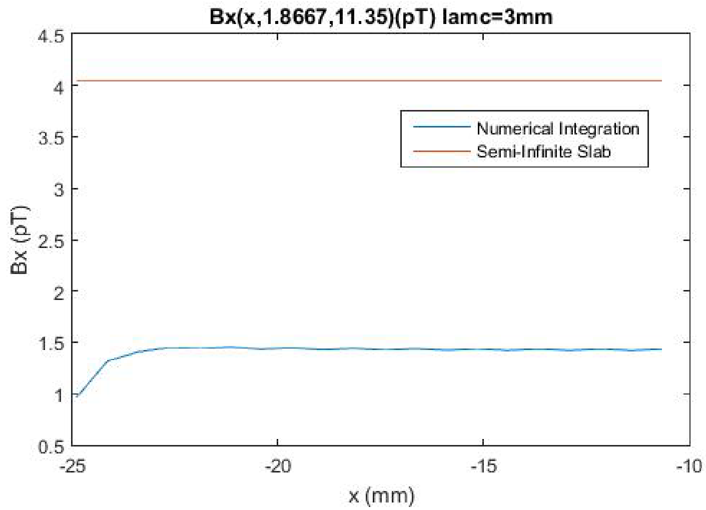


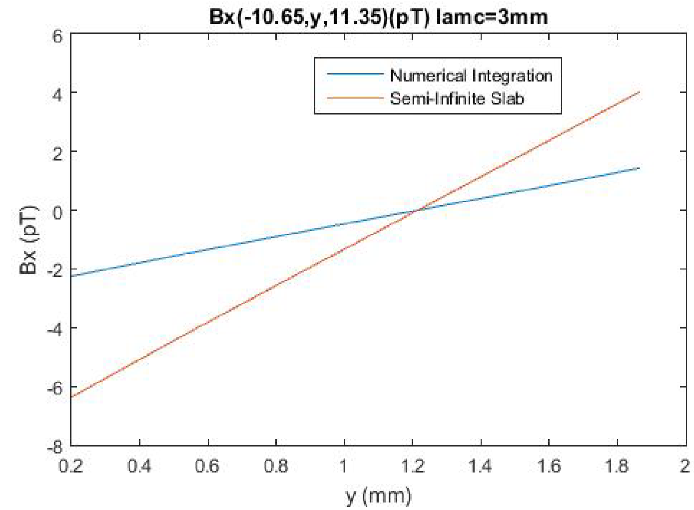


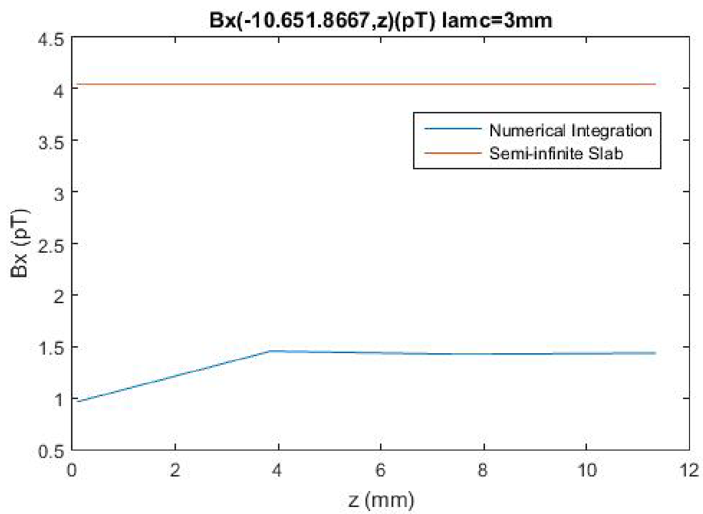
Eight 1-mm thick slabs with 2-mm vacuum gaps
v=[0,0,660m/s], λ_c=1.0mm

Note: Because of slabs above and below the gap, the y-dependence is almost linear rather than exponential.

B_x versus x, y, z



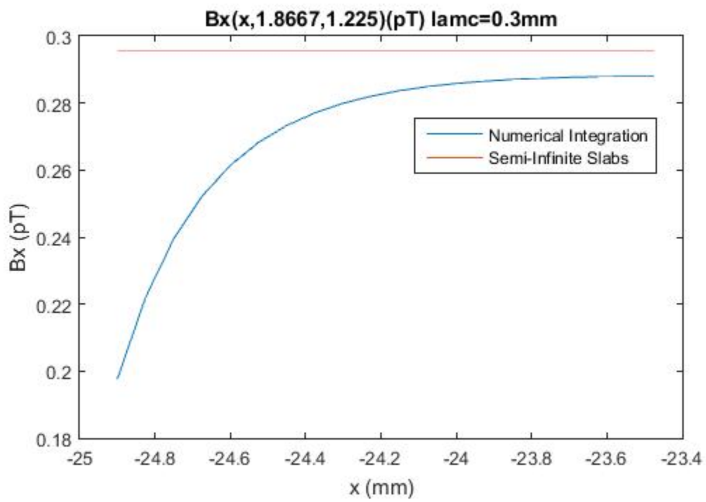


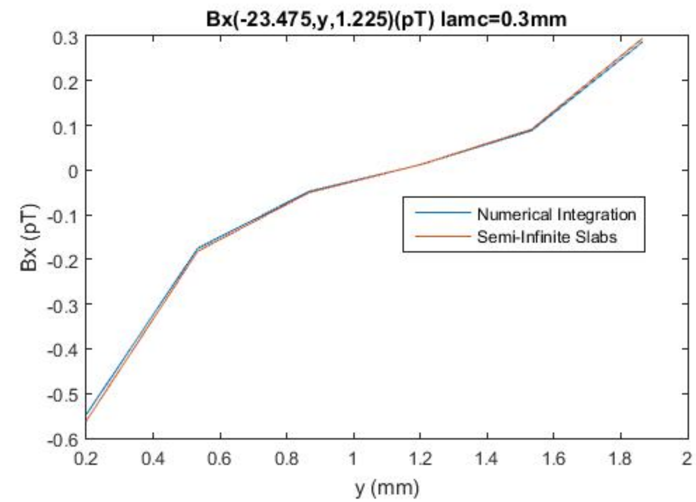


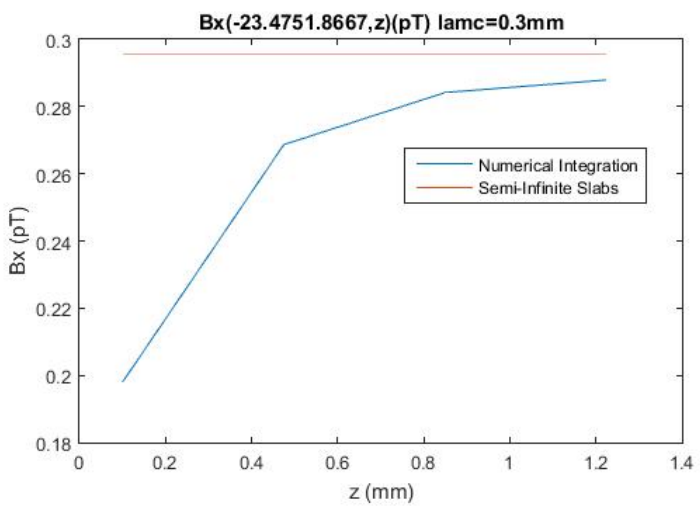
Eight 1-mm thick slabs with 2-mm vacuum gaps

v=[0,0,660m/s], λ_c=3.0mm

B_x versus x, y, z







Eight 1-mm thick slabs with 2-mm vacuum gaps
v=[0,0,660m/s], λ_c=0.3 mm

Note: Since λ_c is 1/3 the thickness of the slab, the slab approximates the semi-infinite slab case in the bulk region away from the edges.

Averaging over volumes in vacuum gap.

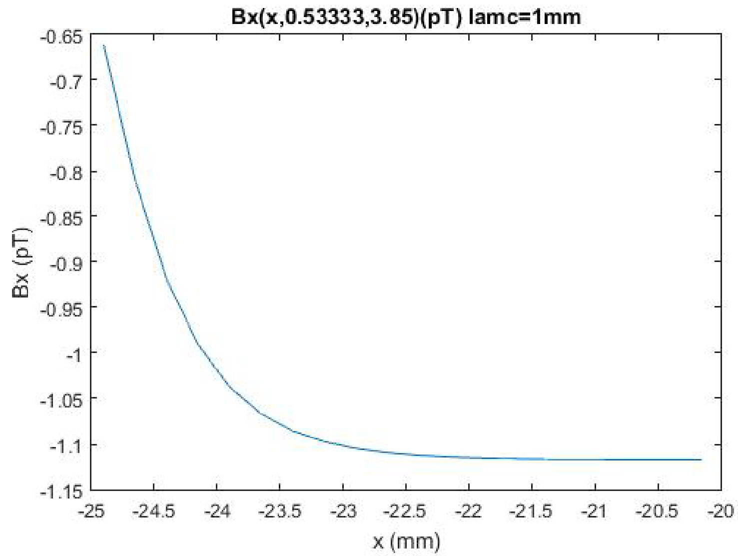
These micro-radian rotations, ϕ, are done separately ($\phi=-\gamma_n BL/v$) for x- and y- components.

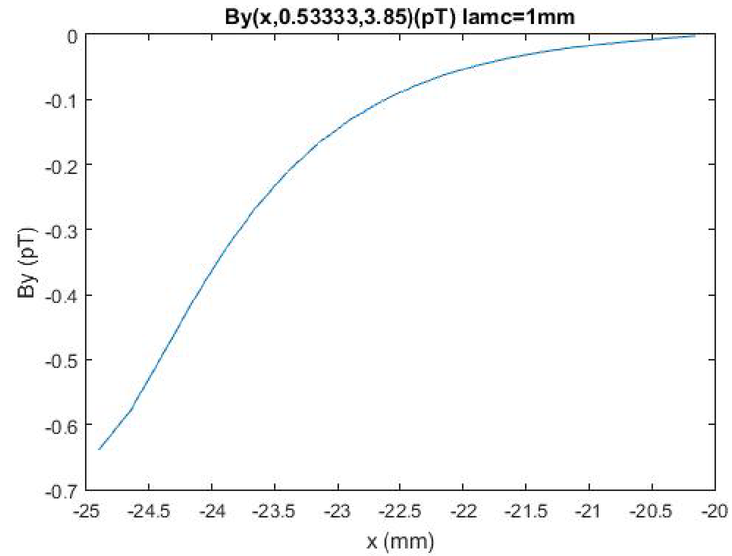
ϕ_x means rotation about B_x…

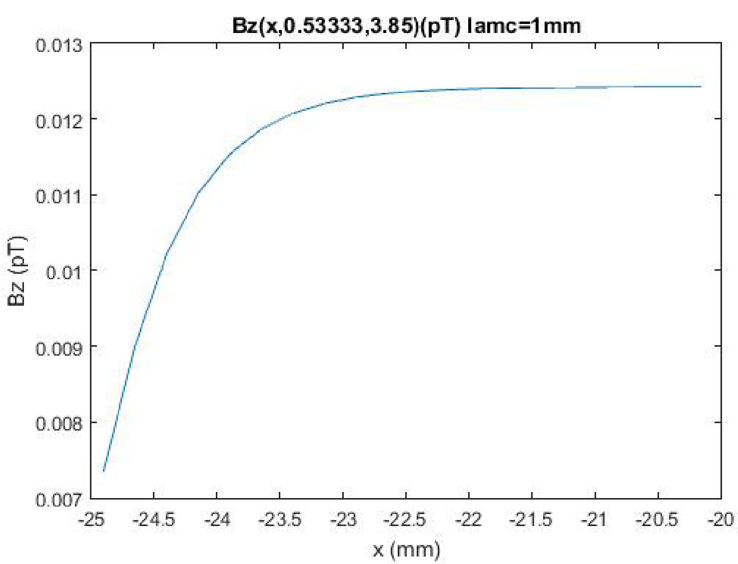
g_A=1E-8 $v=v_z$=660m/s	λ_c=0.3 mm	λ_c=1.0 mm	λ_c=3.0 mm
Avg B_x (pT) corner*	-0.052	-0.26	-0.36
Avg B_x (pT) bulk	-0.064	-0.31	-0.40
Avg B_x (pT) Semi-Inf	-0.066	-0.49	-1.14
Avg B_y (pT) corner	-0.015	-0.15	-0.57
Avg ϕ_x (μrad) bulk	8.8	43	56
Avg ϕ_x (μrad) full	8.6	41	49
Avg ϕ_x (μrad) Semi-Inf	9.0	69	158
Avg ϕ_y (μrad) full	0.2	8.5	95

*corner width = $5\lambda_c$

v_x=7.3m/s, v_z=660m/s (m=0.65 glass), λ_c=1.0 mm



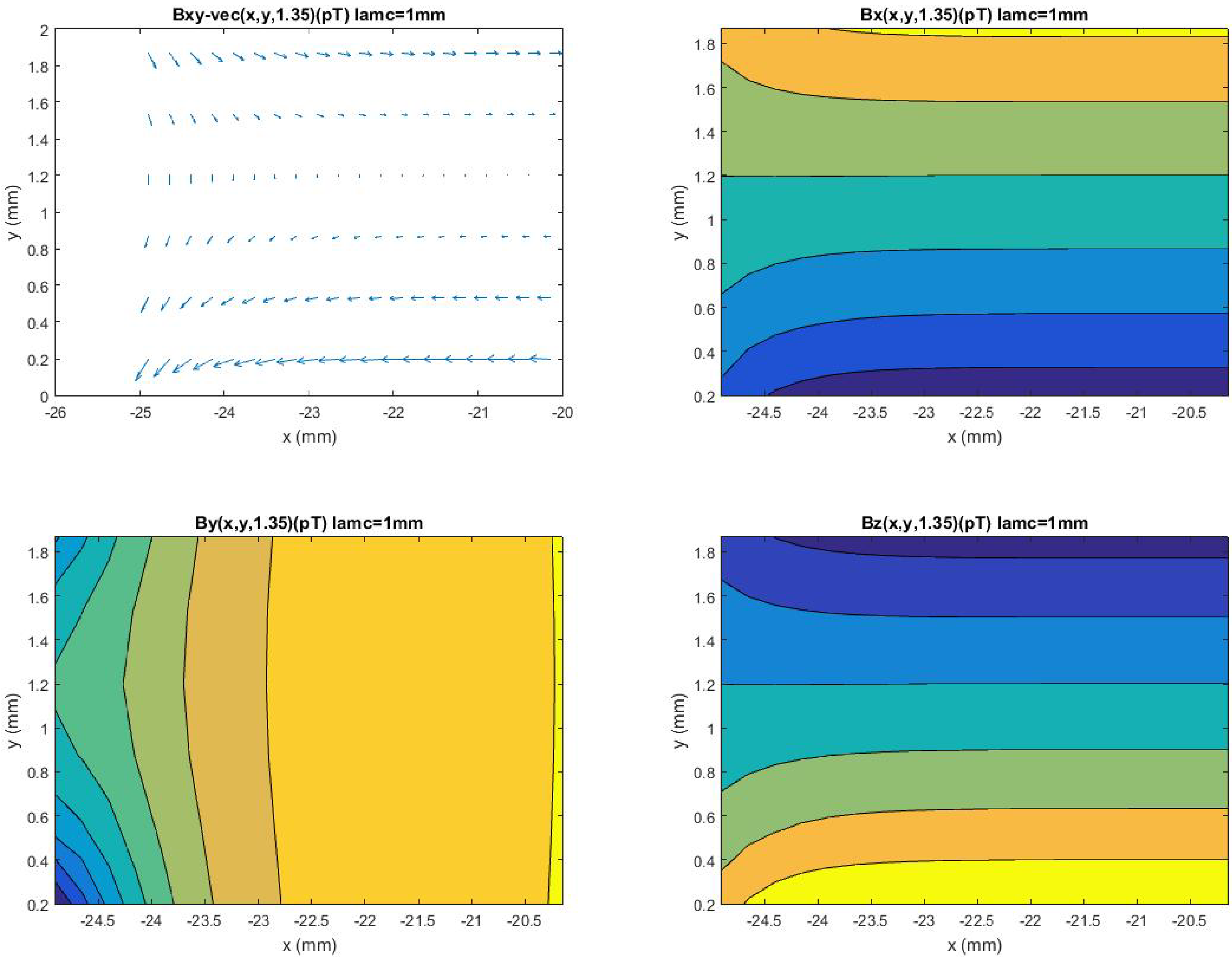




Eight 1-mm thick slabs with 2-mm vacuum gaps
v=[2.6,0,660]m/s, λ_c=1.0 mm

Note: B_x and B_y are essentially unchanged. B_z is quite small.

v_x=7.3m/s, v_z=660m/s (m=0.65 glass), λ_c=1.0 mm



No-bounce acceptance is 2mm/500mm=4mrad, so expect lots of bounces. Angular acceptance from multiple bounces is limited by glass

$$m_{glass} = 0.65, \quad \theta_c^{glass}(10\text{Å}) = 11\text{mrad}$$

v_x=7.3m/s, v_z=660m/s

g_A=1E-8 v_x=7.3m/s v_z=660m/s		λ_c=1.0 mm	λ_c=3.0 mm
Avg B_x (pT) corner*		-0.26	-0.36
Avg B_x (pT) bulk		-0.31	-0.40
Avg B_x (pT) Semi-Inf		-0.49	-1.13
Avg B_y (pT) corner		-0.15	-0.56
Avg B_z (pT) bulk		0.003	0.004
Avg ϕ_x (μrad) bulk		43	55
Avg ϕ_x (μrad) full		40	49
Avg ϕ_x (μrad) Semi-Inf		68	157
Avg ϕ_y (μrad) full		8.3	92
Avg ϕ_z (μrad) full		-0.4	-0.5

So…with maximum divergence, z-rotation component is still quite small.
Y-rotation component is sizable, but beam polarized along y.

Since all rotations are small and if field ~constant along trajectory, treat as one classical rotation of the polarization vector (for μrad, order doesn't matter):

$$\vec{p} = R(\phi_y)R(\phi_z)R(\phi_x)\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-\cos\phi_y \sin\phi_z \cos\phi_x + \sin\phi_y \sin\phi_x\\ \cos\phi_z \cos\phi_x\\ \sin\phi_y \sin\phi_z \cos\phi_x + \cos\phi_y \sin\phi_x\end{bmatrix} \approx \begin{bmatrix}\sin\phi_z\\1\\ \sin\phi_x\end{bmatrix}$$

$\vec{p} \approx \begin{bmatrix}\phi_z\\1\\ \phi_x\end{bmatrix}$; where ϕ_z is at most 1% of ϕ_x.

The z-rotation is very small for even most divergent neutrons and since we start polarized in y-direction and y-rotation is also small, the net rotation is essentially only dependent on rotation from B_x. So, the simulation code can focus only on size of $B_x(x, y, z)$ in vacuum gap region.

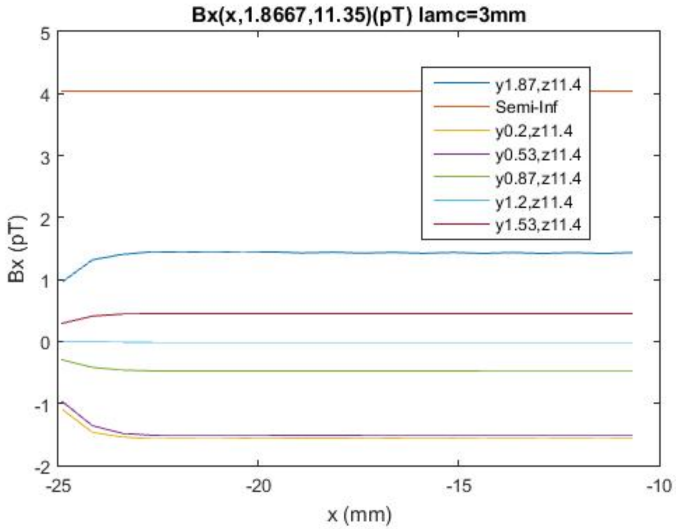
Field is not constant. How to deal with in simulation (nSimDark)…

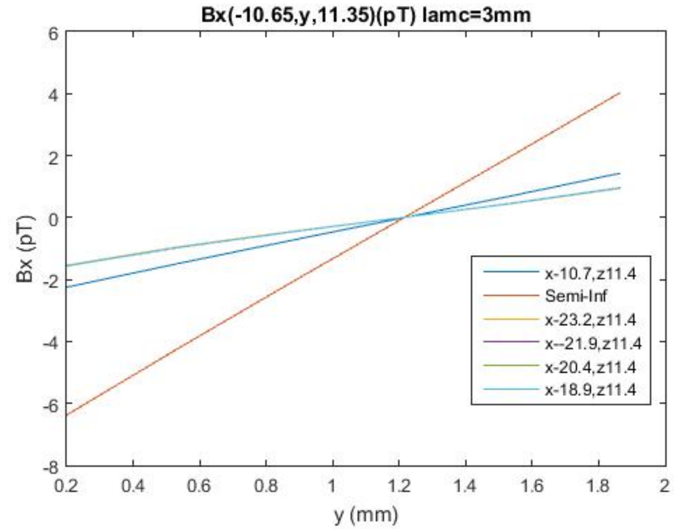
- Fringing on front and back (z): small ~4mm/500mm<1% effect →Ignore
- Fringing on sides (x): not too small, 1—10% depending on λ_c → scale field size by an average of polynomial fit of $B_x(x)$ within x-range of trajectory if trajectory is mostly in side region.
- Moving through different y-values, field changes significantly. Use analytic integration of polynomial fit of $B_x(y)$ along trajectory to determine ϕ.

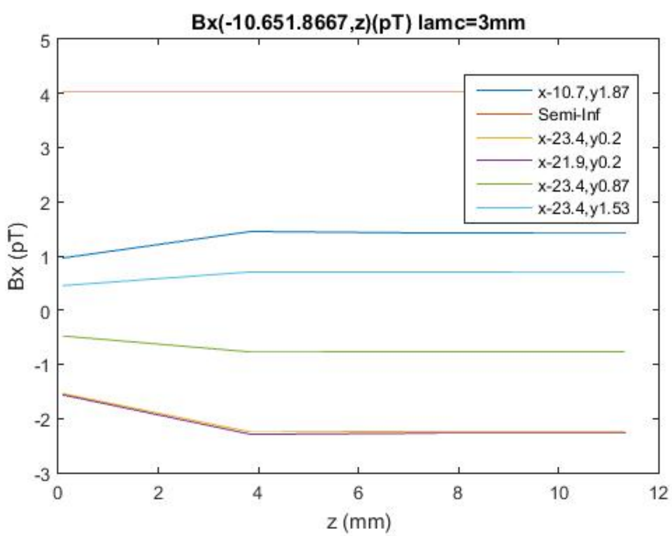
For example, for $\lambda_c = 1.0\text{mm}$ $B_x(y) = 0.28y^3 - y^2 + 2.8y - 2.4$ (pT) for $y[0,2]$,
where $y = y_o + \frac{(y_f - y_i)}{(z_f - z_i)}(z - z_i)$

So

$$\phi = \frac{-\gamma_n}{v_n}\int_{z_i}^{z_f} B_x[y(z)]dz = \ldots$$


Bx(x,1.8667,11.35)(pT) lamc=3mm
y1.87,z11.4
Semi-Inf
y0.2,z11.4
y0.53,z11.4
y0.87,z11.4
y1.2,z11.4
y1.53,z11.4
Bx (pT)
x (mm)


Bx(-10.65,y,11.35)(pT) lamc=3mm
x-10.7,z11.4
Semi-Inf
x-23.2,z11.4
x--21.9,z11.4
x-20.4,z11.4
x-18.9,z11.4
Bx (pT)
y (mm)


Bx(-10.651.8667,z)(pT) lamc=3mm
x-10.7,y1.87
Semi-Inf
x-23.4,y0.2
x-21.9,y0.2
x-23.4,y0.87
x-23.4,y1.53
Bx (pT)
z (mm)